Data-driven Decision-Making in Dynamic Environments

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Outline

- Anomaly Detection in Dynamic Environment
- Data-driven Decision-Making

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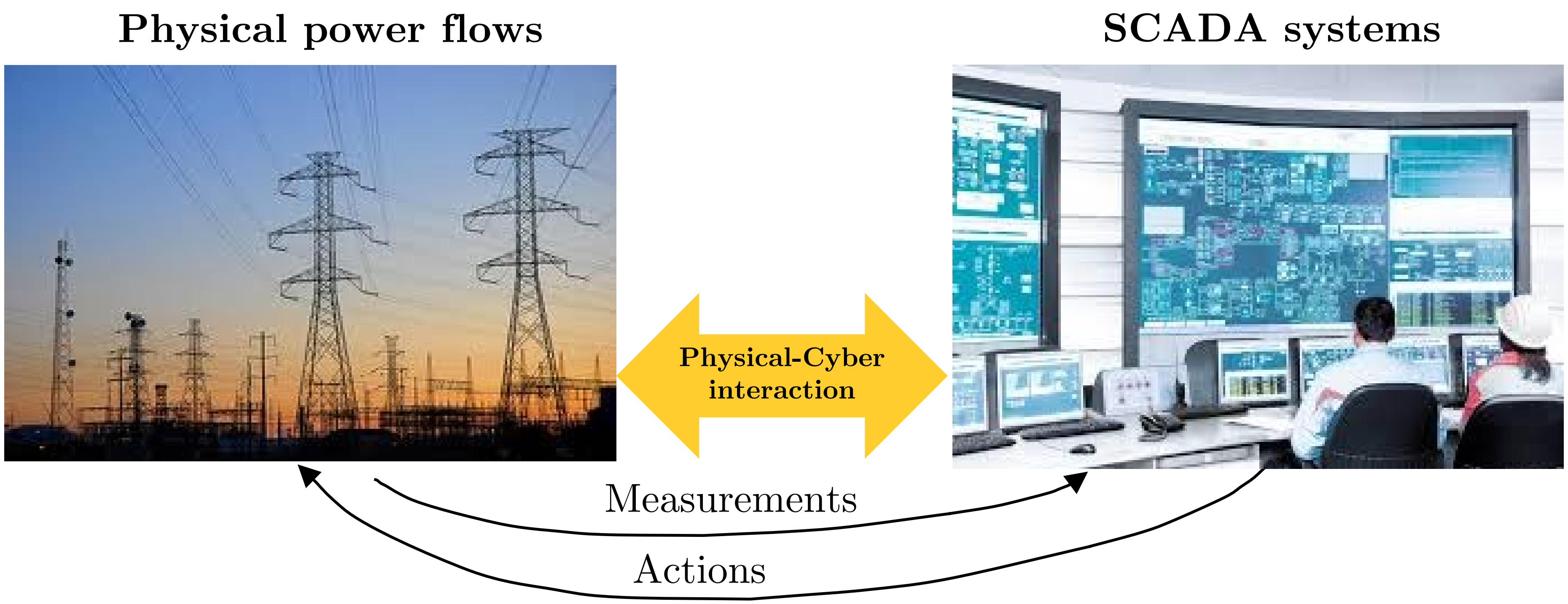
- Anomaly Detection in Dynamic Environment
  - Problem statement
  - Model description
  - Optimization-based solution

Motivation

Electric power system

- ✓ Uninterrupted energy supply: **vital** for many critical infrastructures
- ✓ **Susceptible** to operational errors and external attacks
- ✓ **Digitalization** makes the system even more vulnerable



Motivation

Electric power system

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- ✓ **Susceptible** to operational errors and external attacks
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Physical power flows

ATTACK

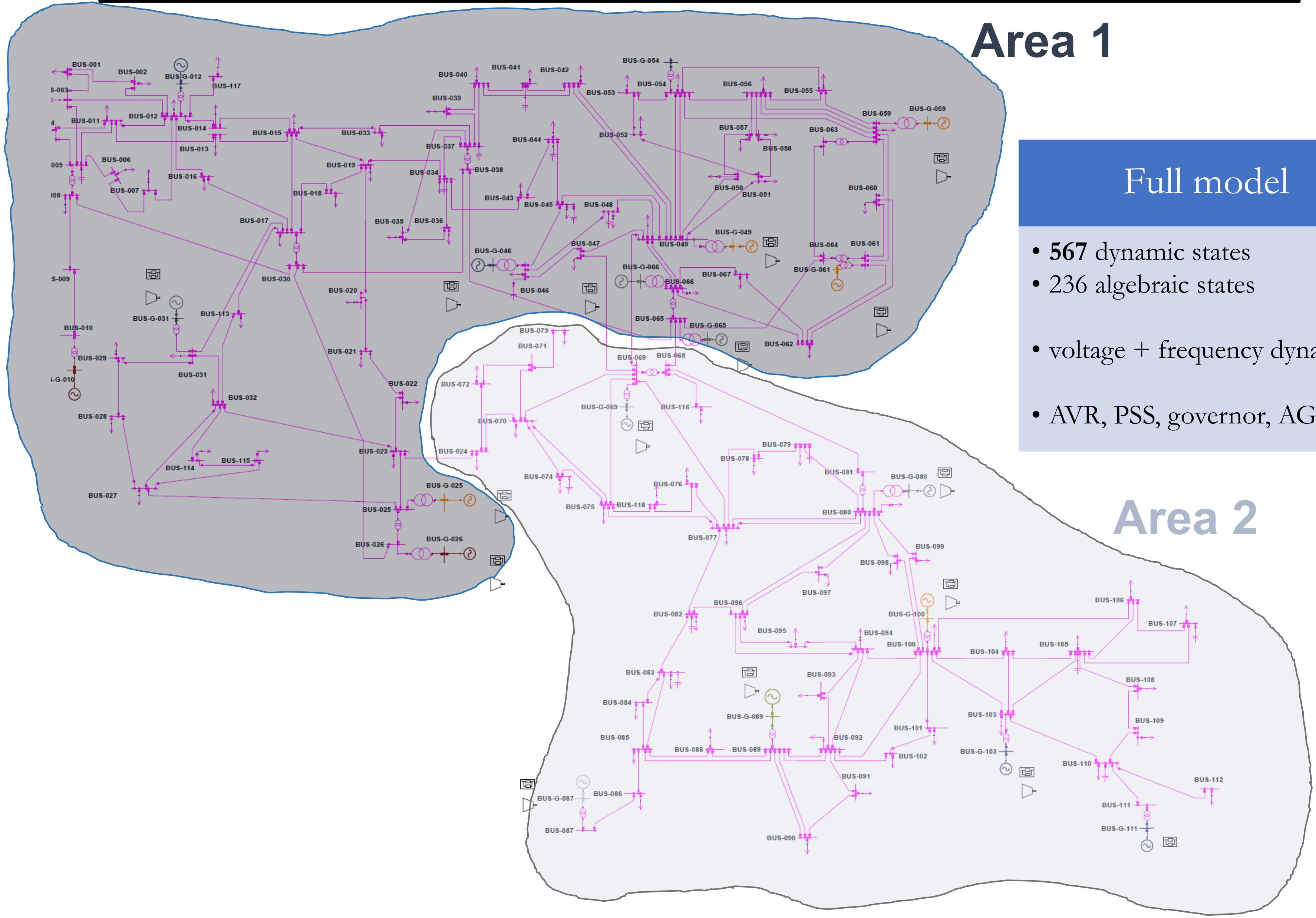
Physical-Cyber interaction

SCADA systems

Measurements

Actions

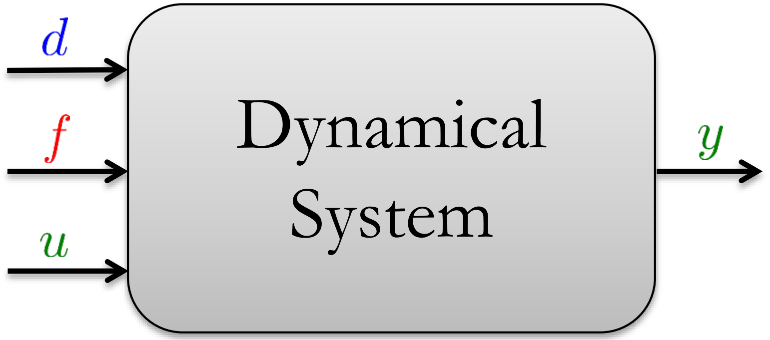
Motivated by Automatic Generation Control (AGC) Case Study



Full model

- **567** dynamic states
- 236 algebraic states
- voltage + frequency dynamics
- AVR, PSS, governor, AGC

Problem Statement



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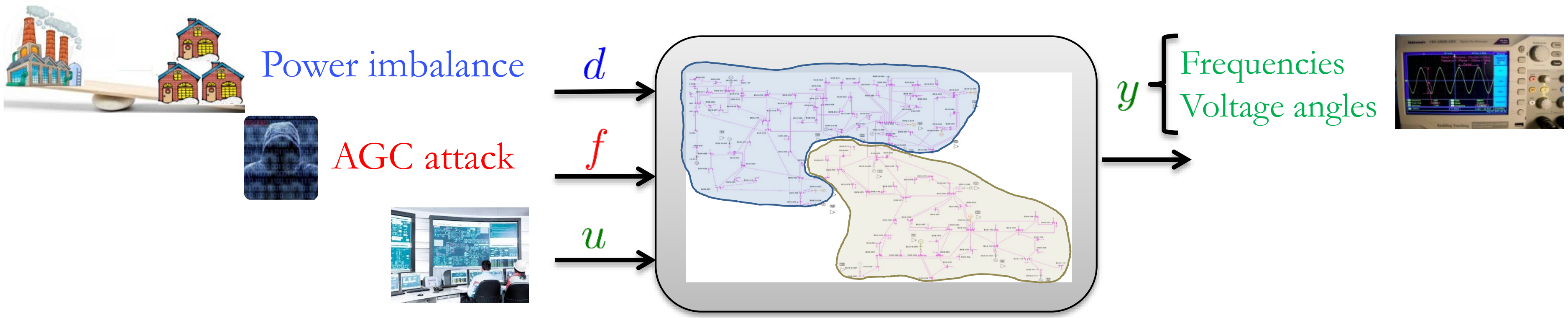

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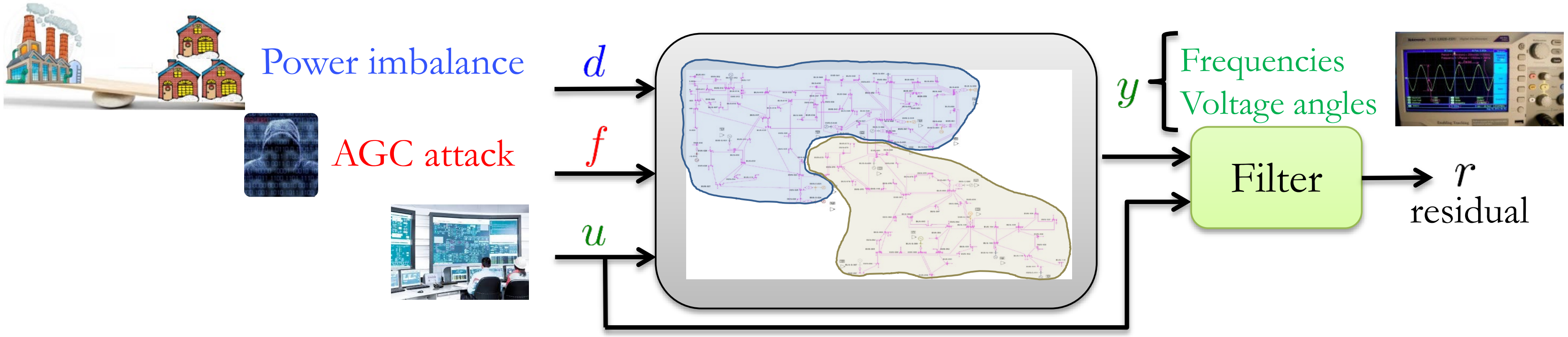

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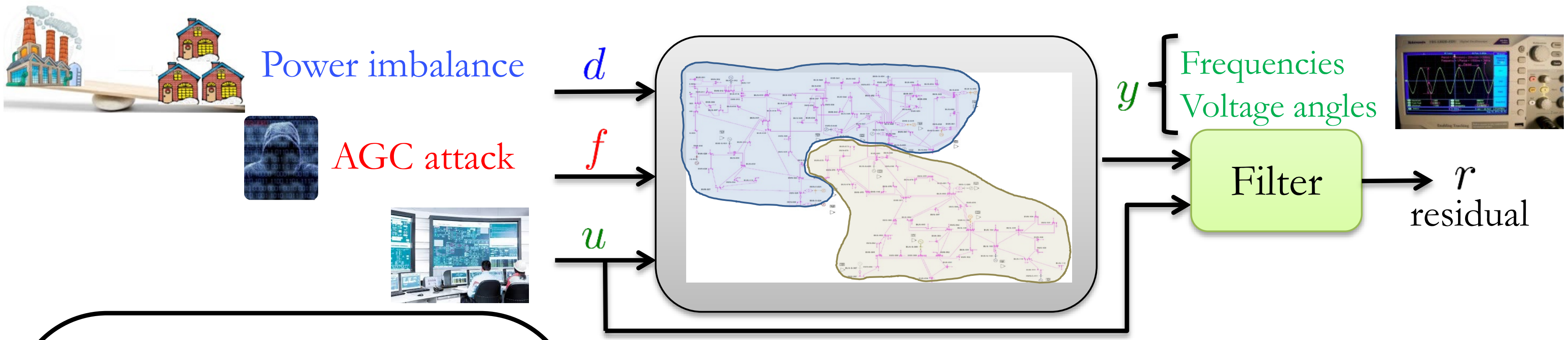

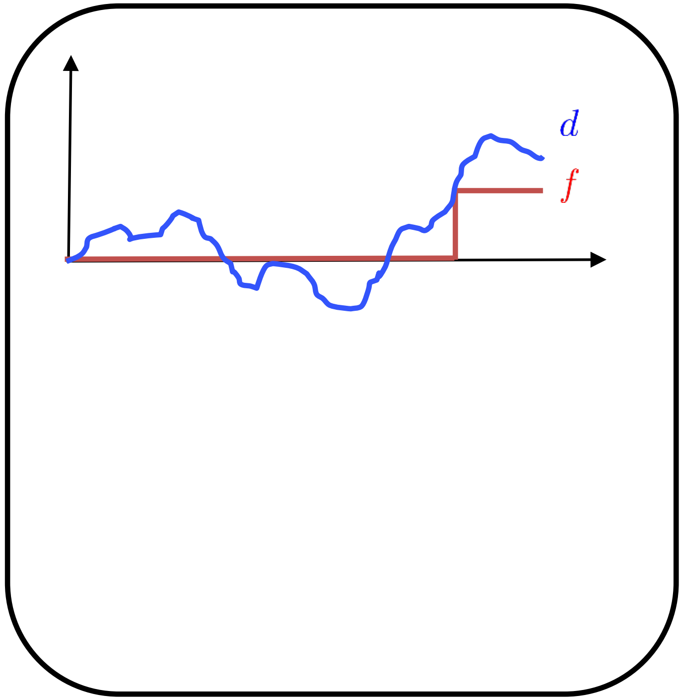

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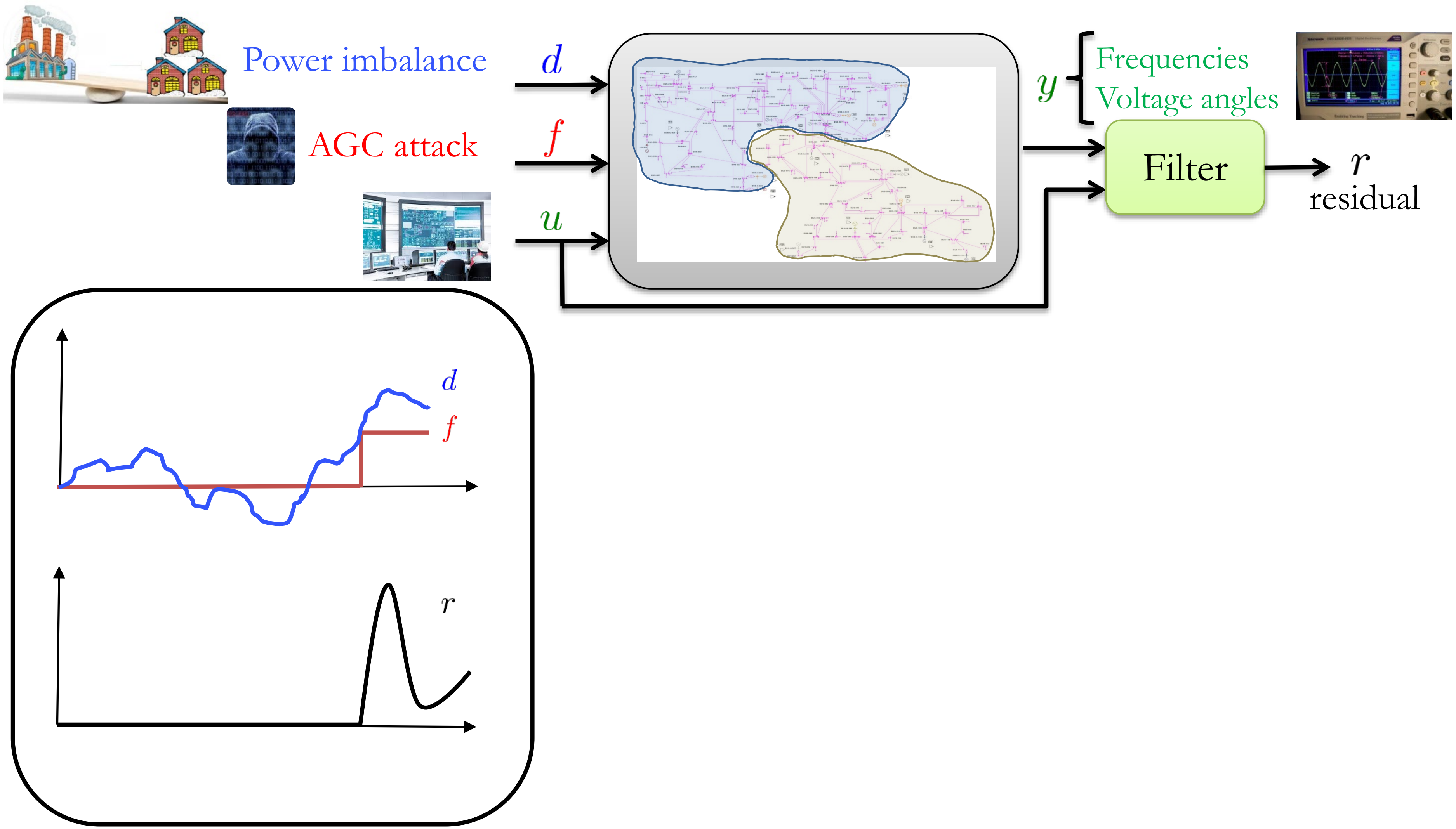

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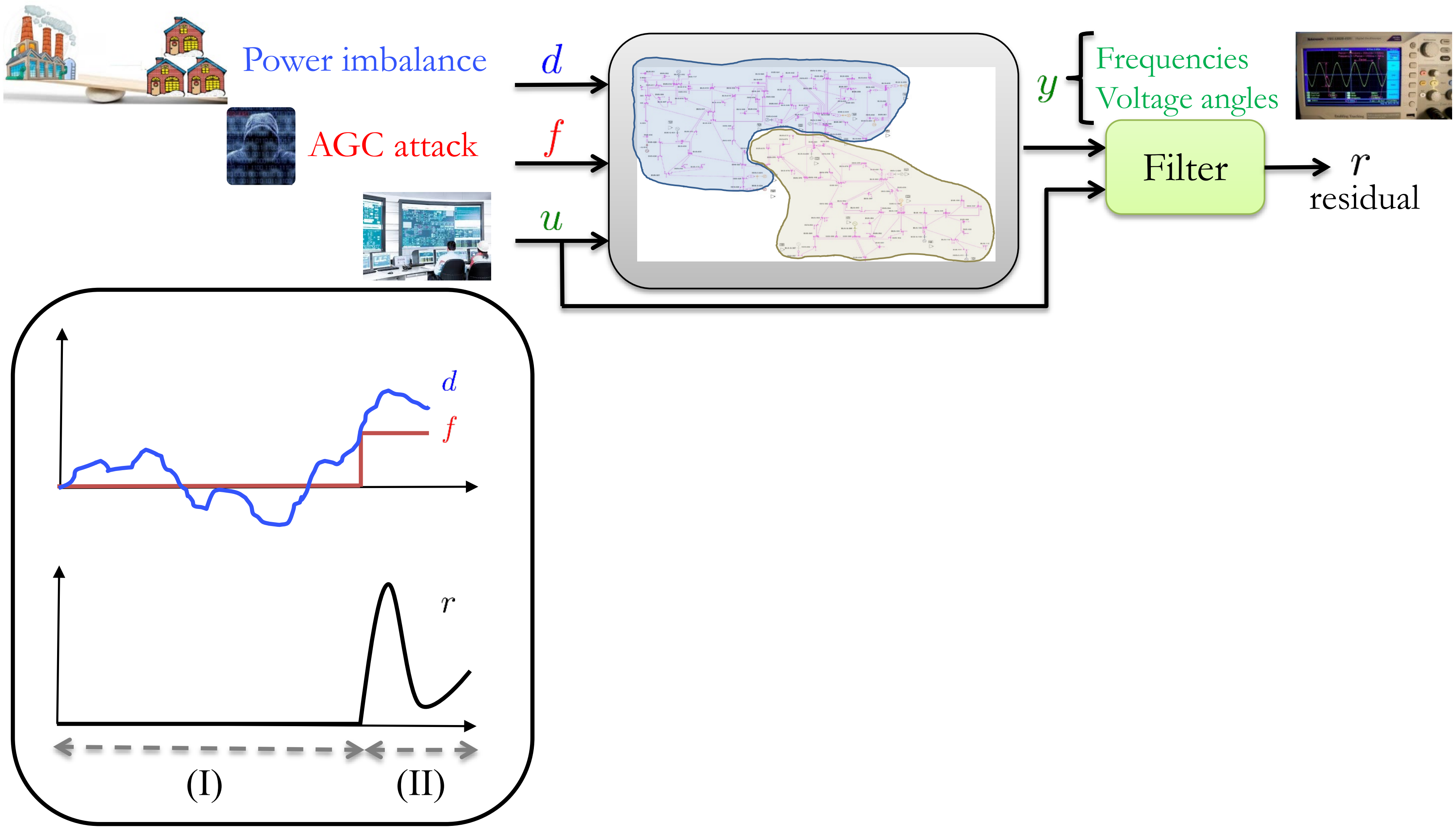

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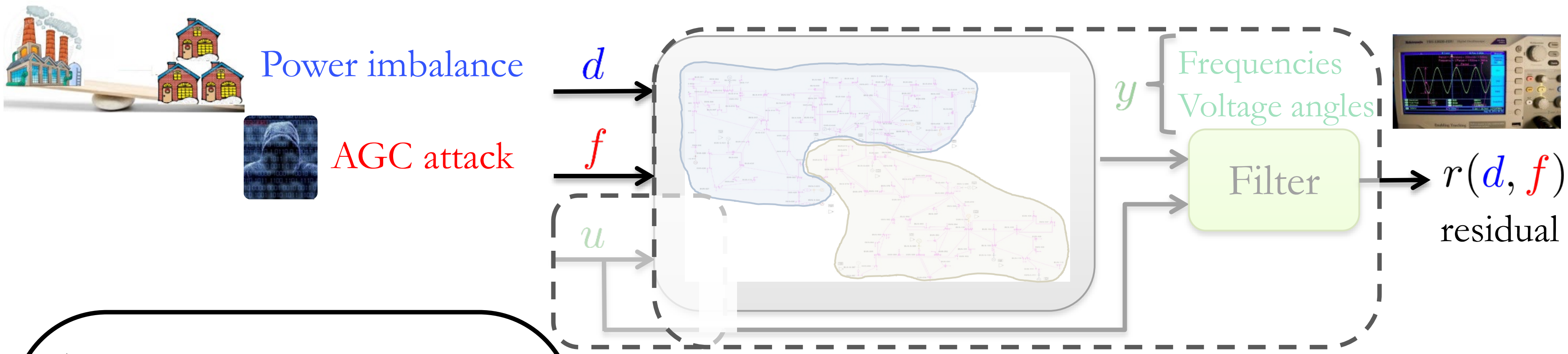

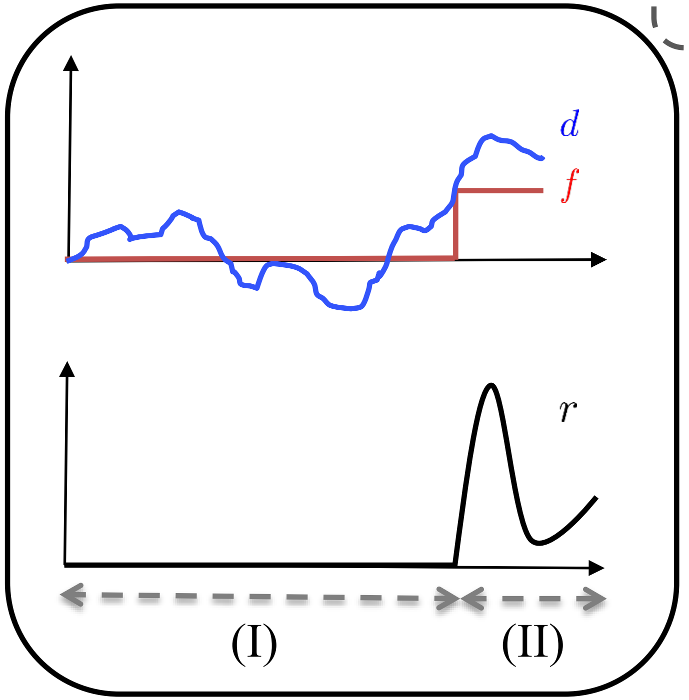

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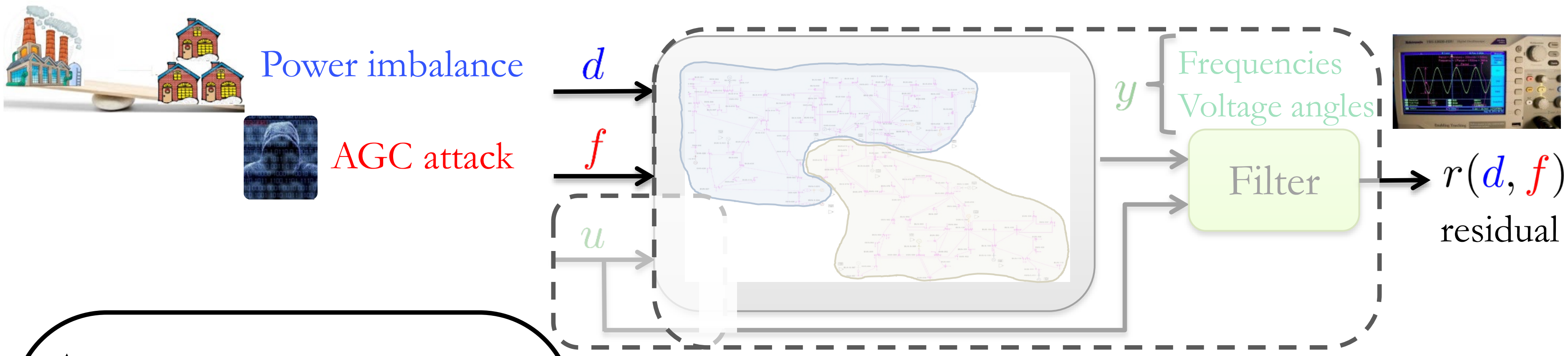
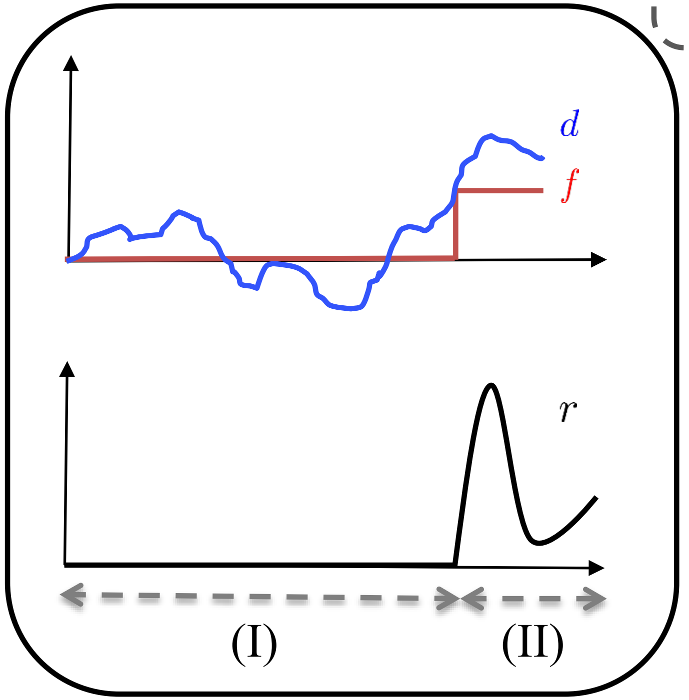


Fault Detection $\begin{cases} \text{(I)} \quad d \mapsto r(d,0) \equiv 0 \\ \text{(II)} \quad f \mapsto r(d,f) \neq 0, \quad \forall d \end{cases}$



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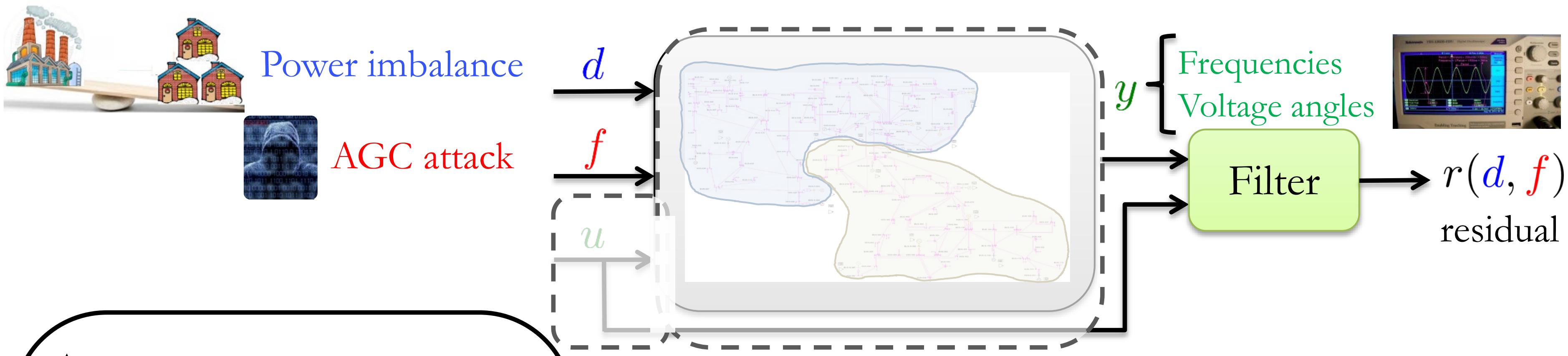

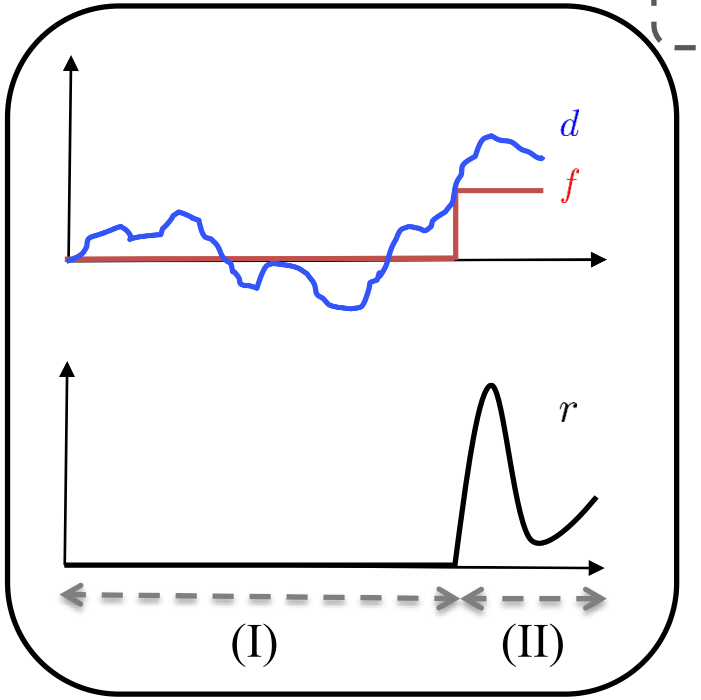

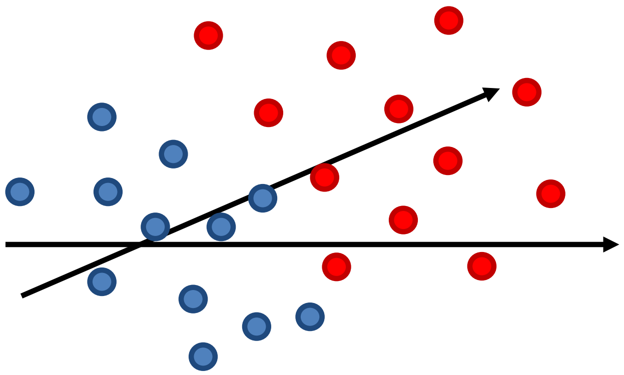

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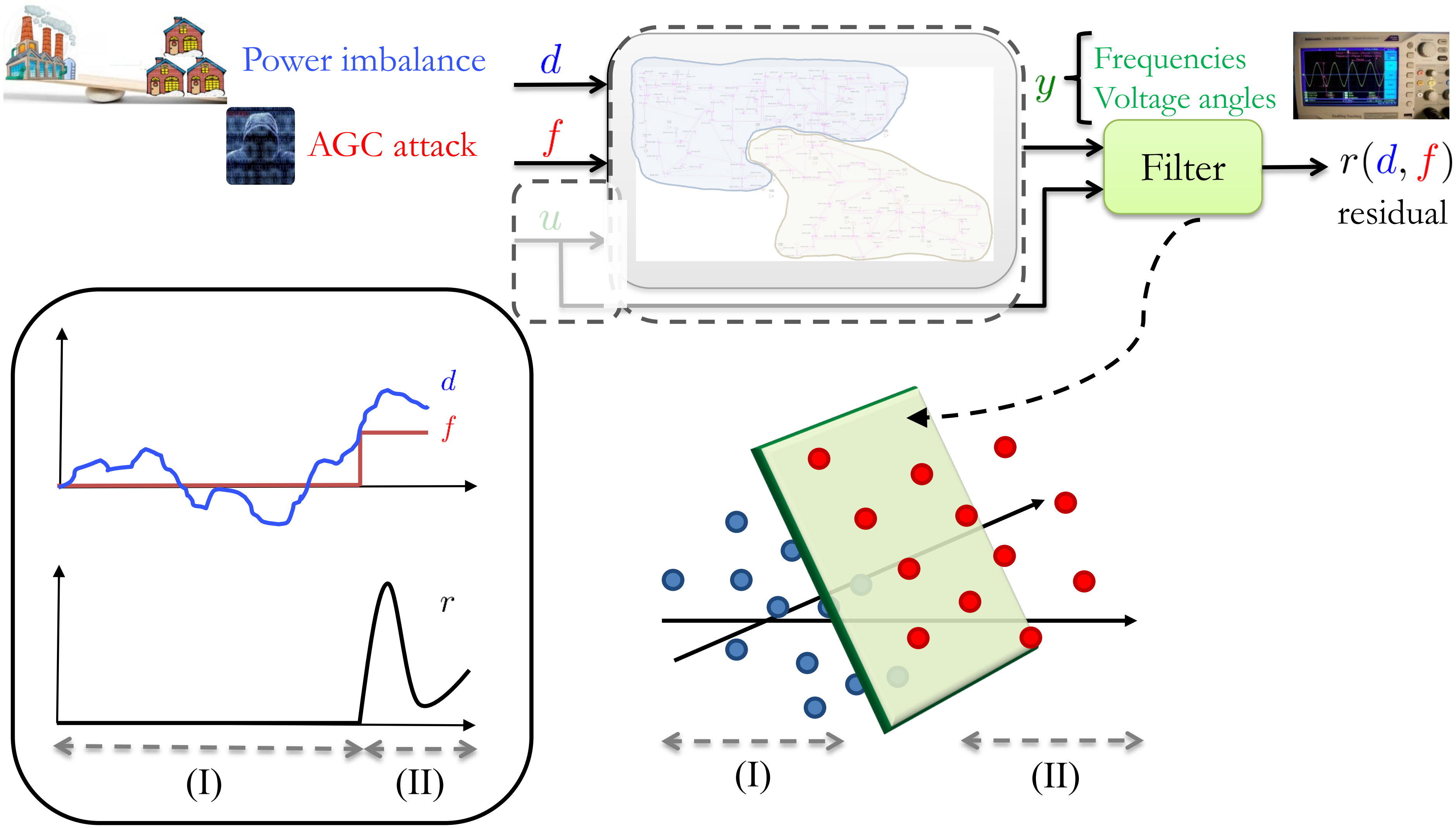

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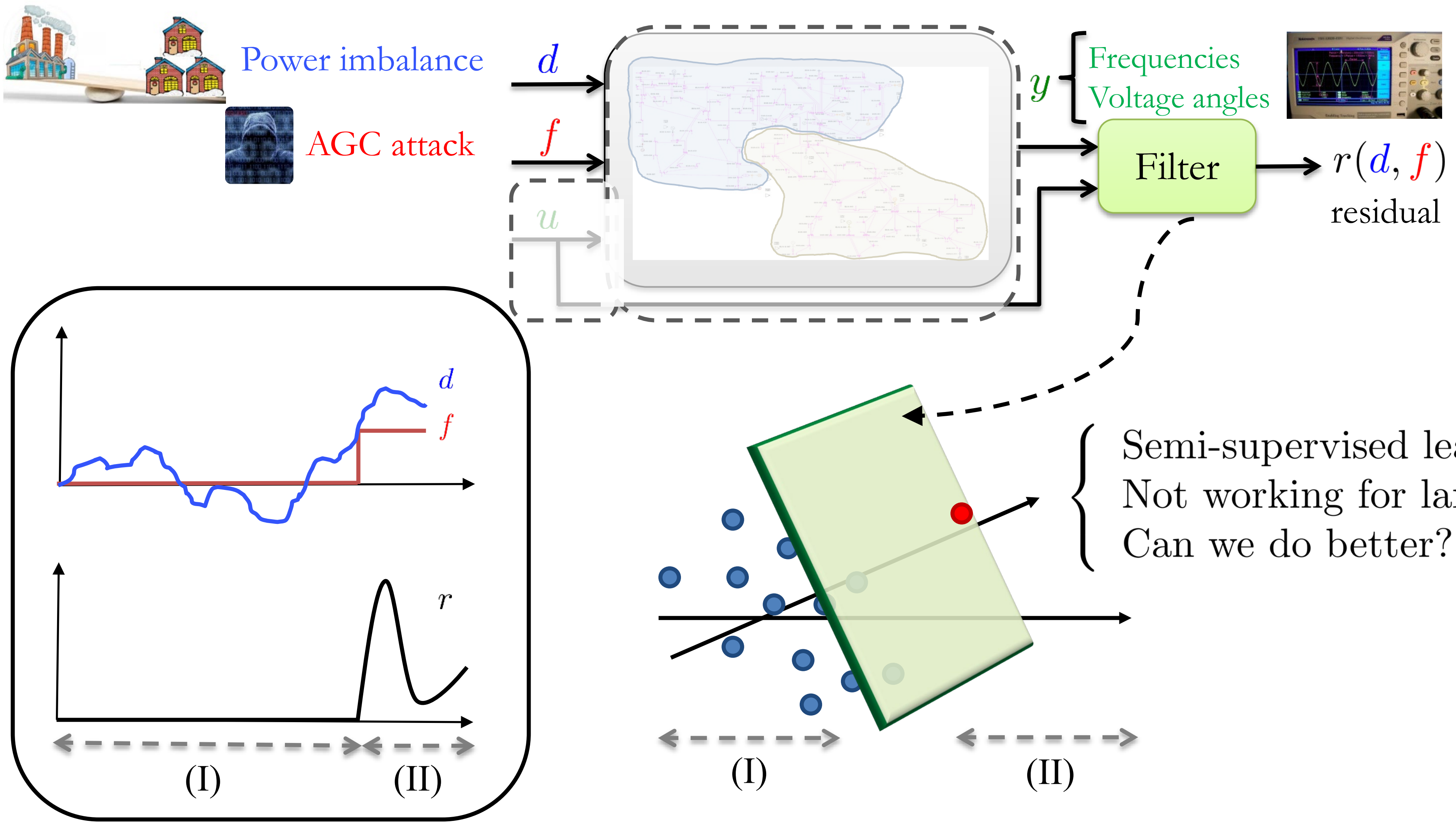
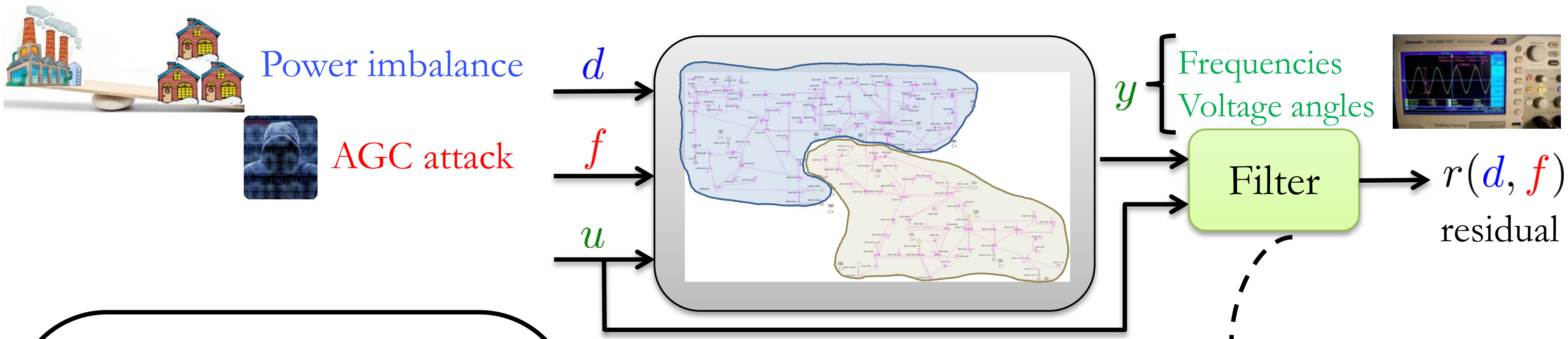
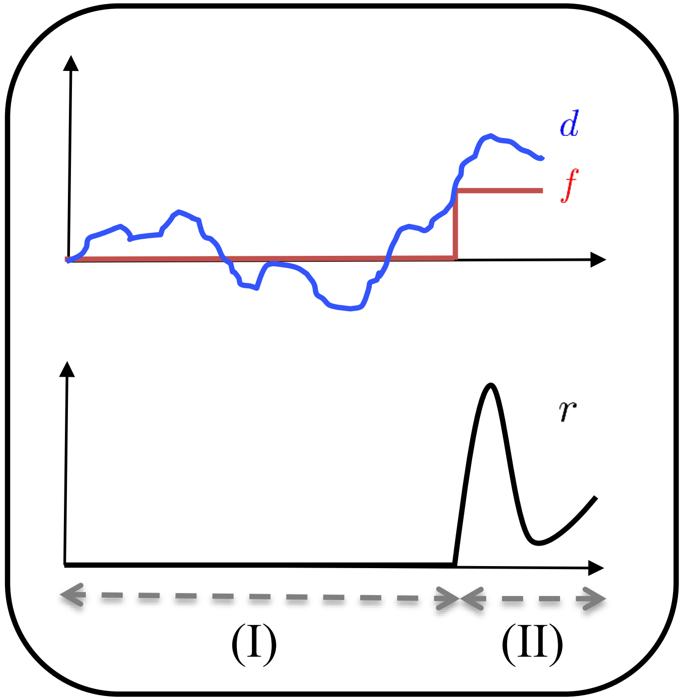
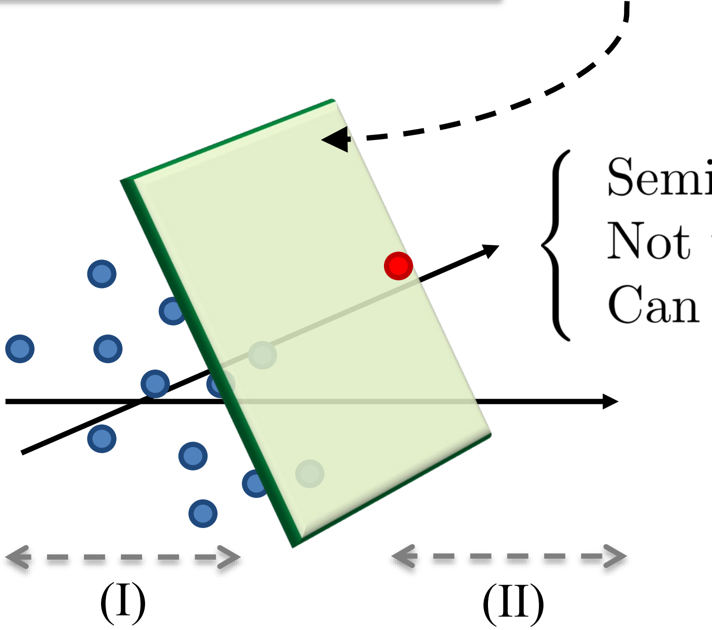


Semi-supervised learning
Not working for large-scale
Can we do better?

Problem Statement







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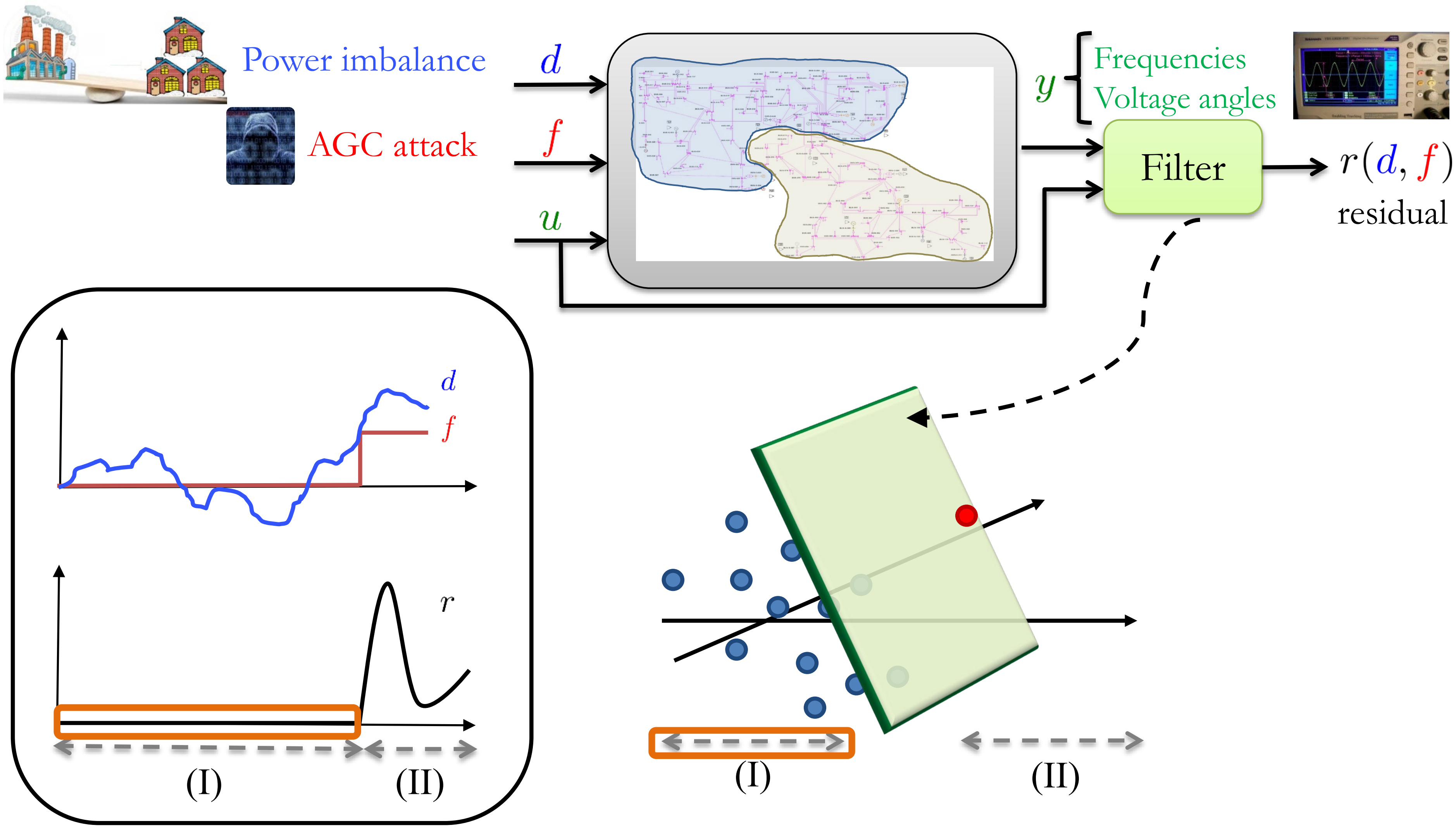

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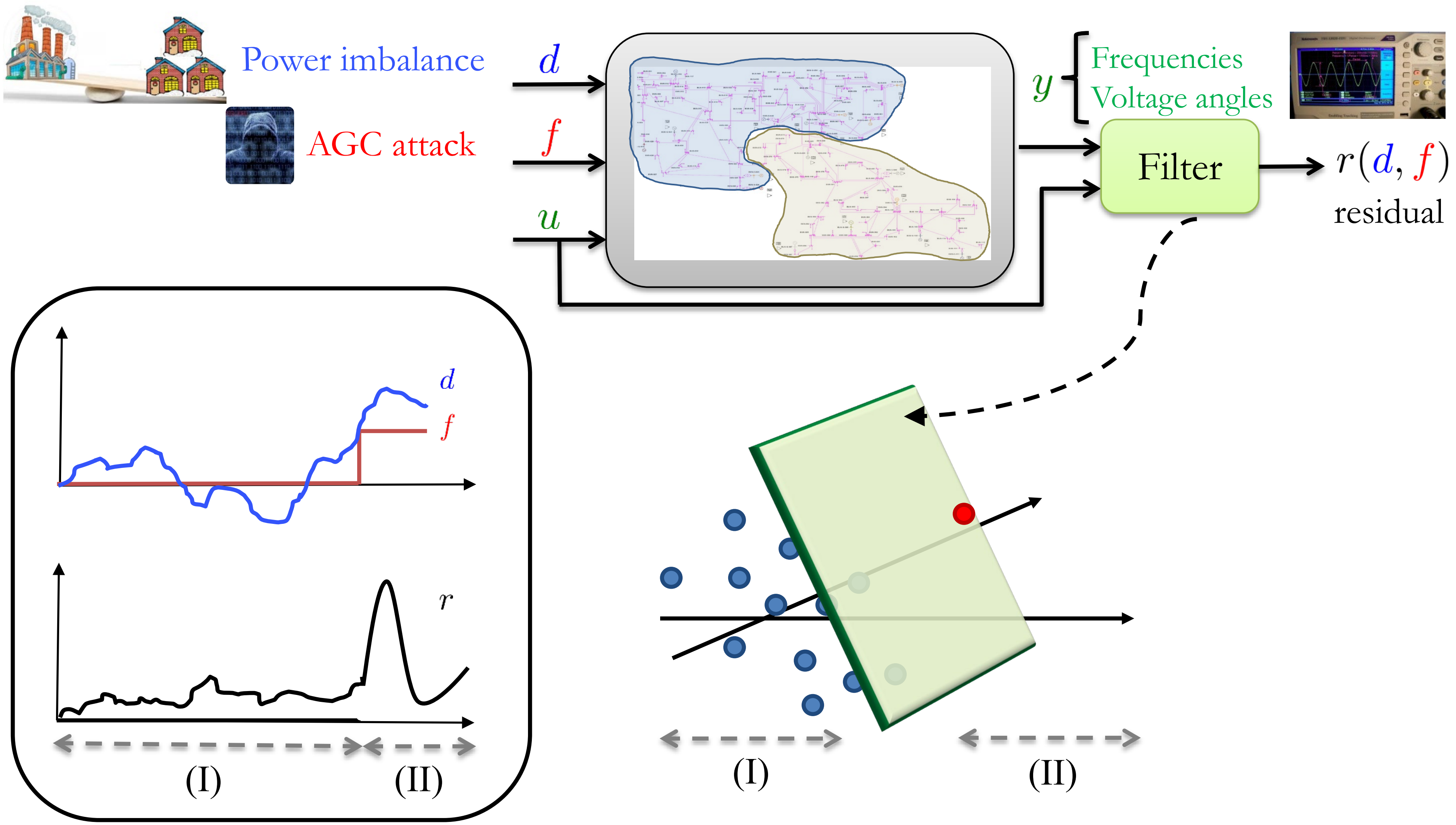

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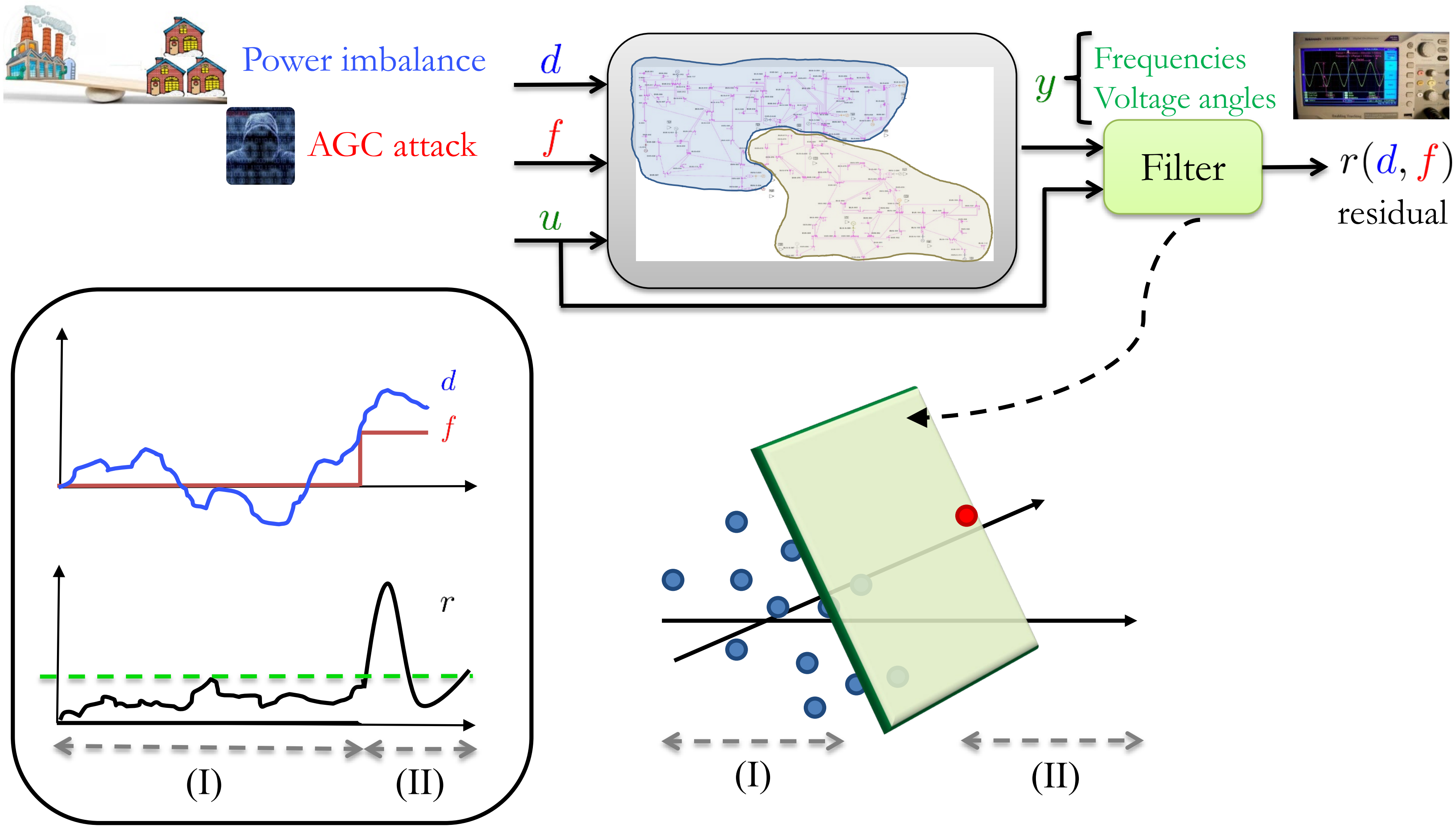

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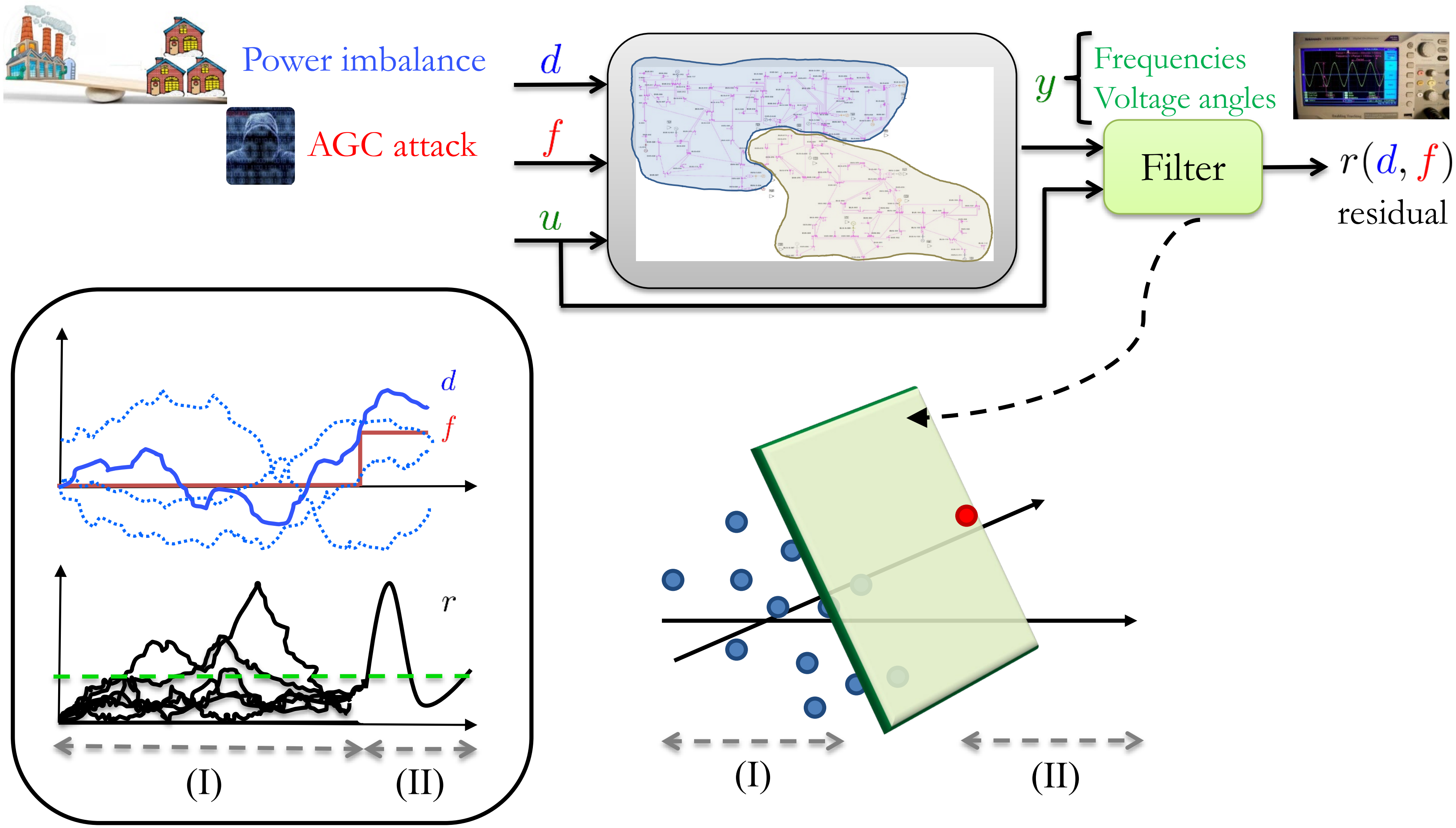

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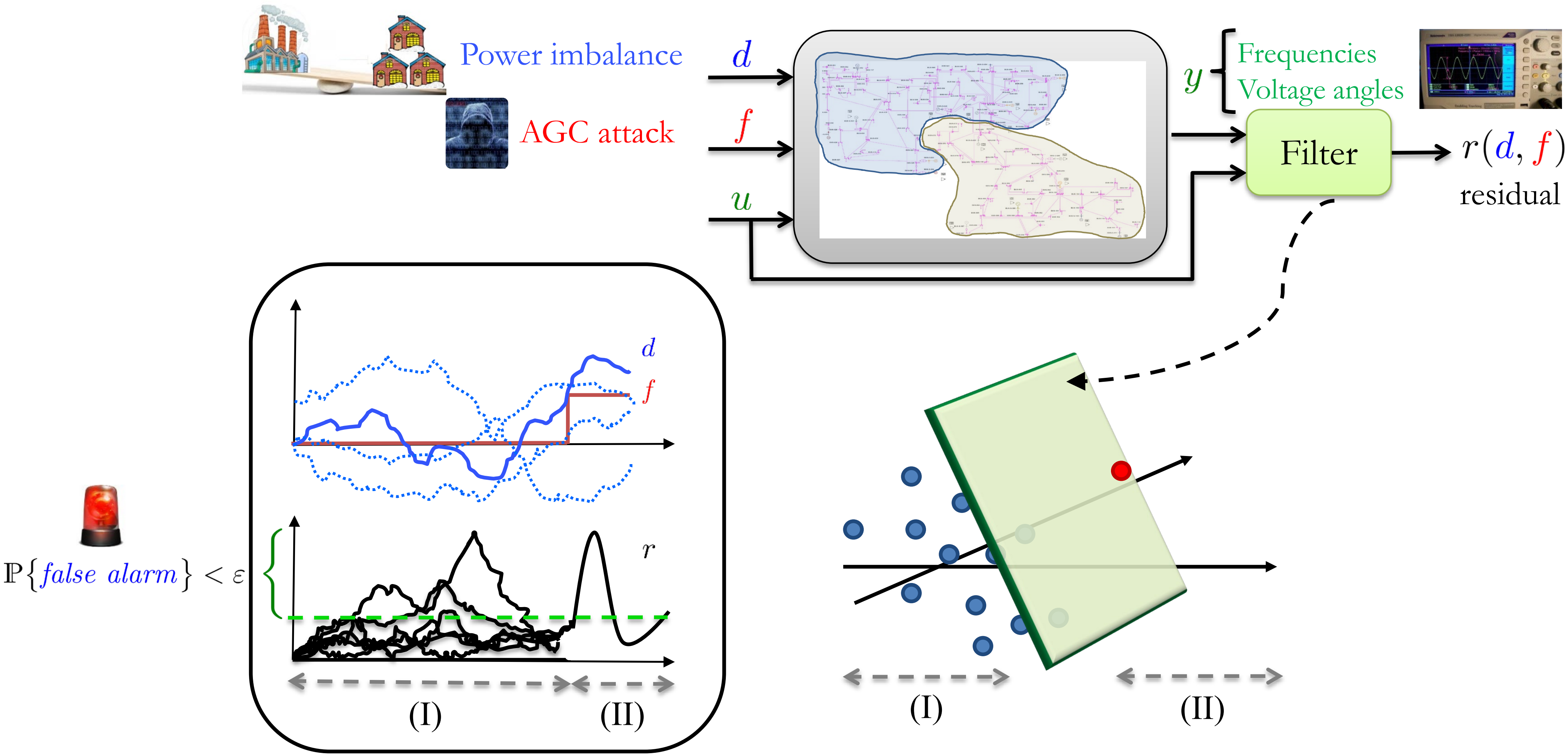

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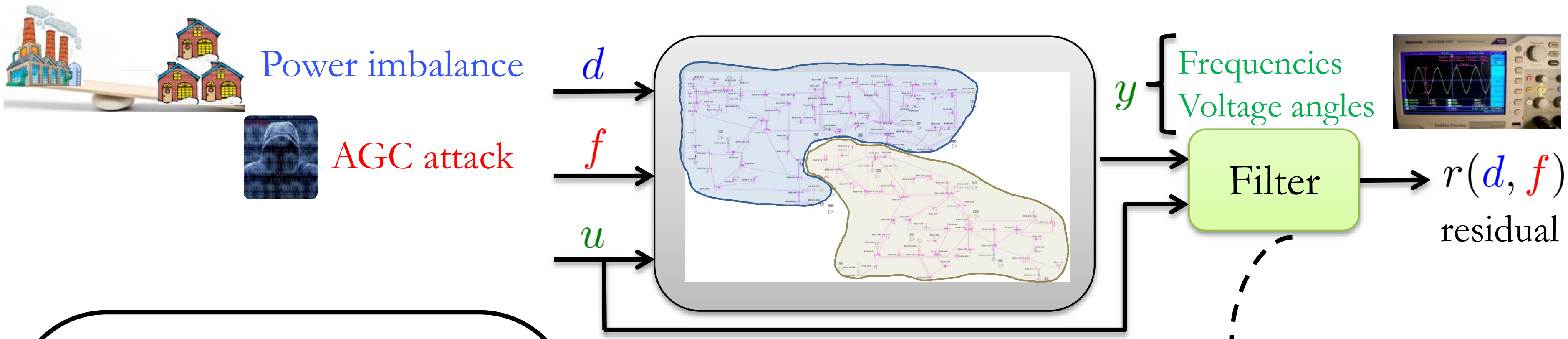

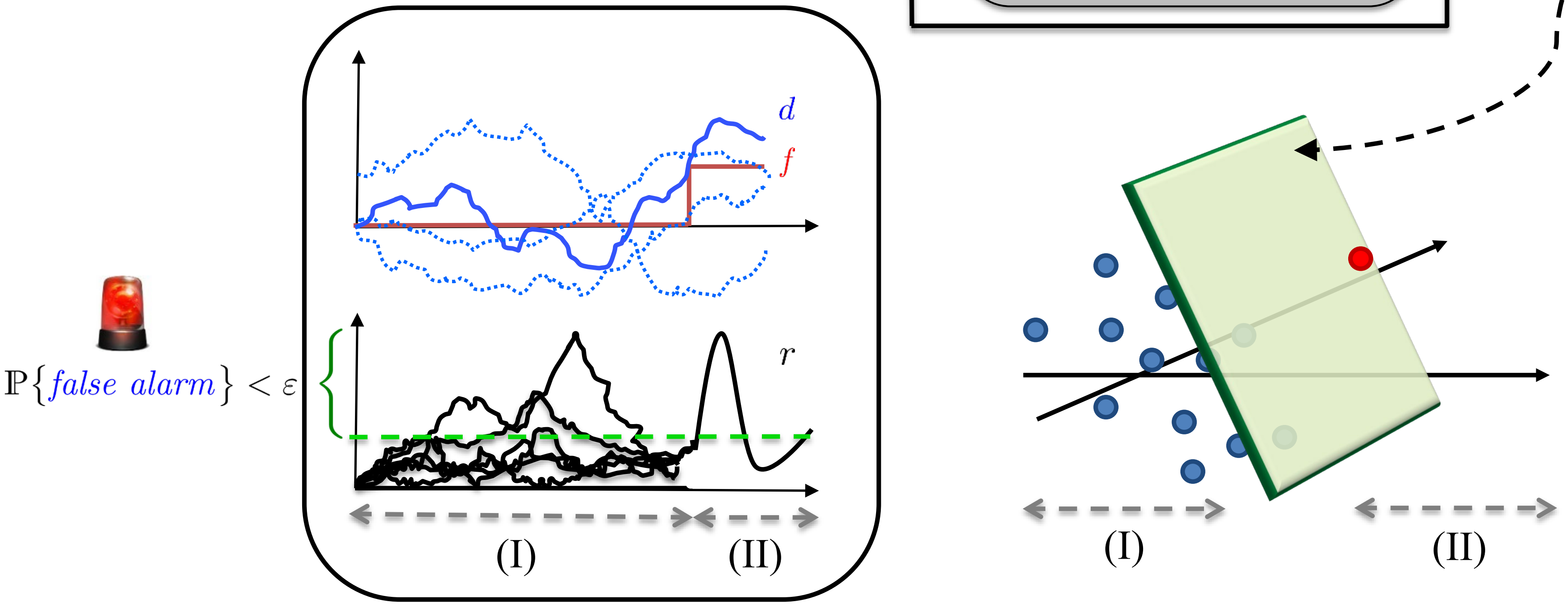

Design parameters:

(i) Filter

(ii) Threshold

Sources of difficulty:

(i) Nolinear dynamics

(ii) Unknown disturbance

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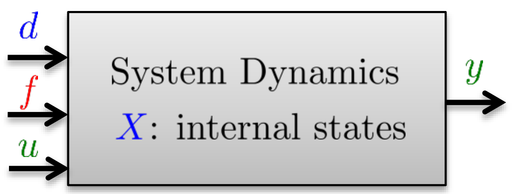
Model Description

$$\begin{cases} X^{+} &= AX + B_u u + B_d d + B_f f + E_X(X, d) \\ y &= CX + D_u u + D_d d + D_f f + E_Y(X, d) \end{cases}$$



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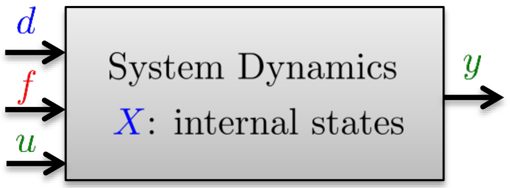

Discrete-time dynamics

$$\begin{cases} X(t+1) &= AX(t) + B_u u(t) + B_d d(t) + B_f f(t) + E_X\big(X(t), d(t)\big) \\ y(t) &= CX(t) + D_u u(t) + D_d d(t) + D_f f(t) + E_Y\big(X(t), d(t)\big) \end{cases}$$

Continuous-time dynamics

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t} X(t) &= AX(t) + B_u u(t) + B_d d(t) + B_f f(t) + E_X\big(X(t), d(t)\big) \\ y(t) &= CX(t) + D_u u(t) + D_d d(t) + D_f f(t) + E_Y\big(X(t), d(t)\big) \end{cases}$$

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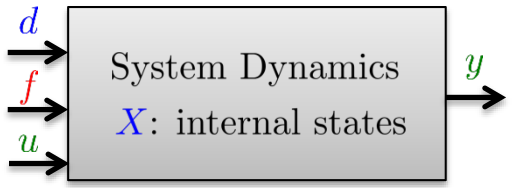

$q := \begin{bmatrix} X \\ d \end{bmatrix}$ → Unknown signals (disturbances & internal states)

$z := \begin{bmatrix} y \\ u \end{bmatrix}$ → Known signals (state measurements & control signals)

f := fault → Intrusion, fault signal, physical damage

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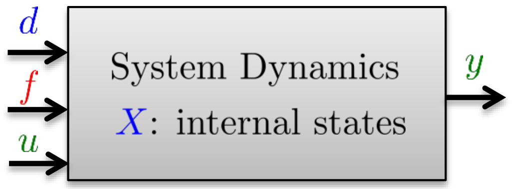

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$$E(q) = \begin{bmatrix} E_X(X, d) \\ E_Y(X, d) \end{bmatrix}, \quad H(p) = \begin{bmatrix} -pI + A & B_d \\ C & D_d \end{bmatrix}, \quad L = \begin{bmatrix} 0 & B_u \\ -I & D_u \end{bmatrix}, \quad F = \begin{bmatrix} B_f \\ D_f \end{bmatrix}$$

$$\boxed{E(q) + H(p)q + L(p)z + F(p)f = 0 \quad \text{Differential-Algebraic Equations (DAE)}}$$

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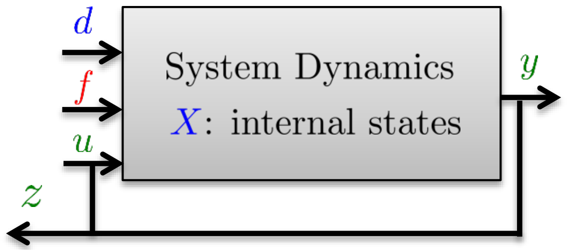
Filter Construction

$$\underbrace{E(q)}_{\text{Nonlinear term}} + H(p)q + L(p)z + F(p)f = 0$$



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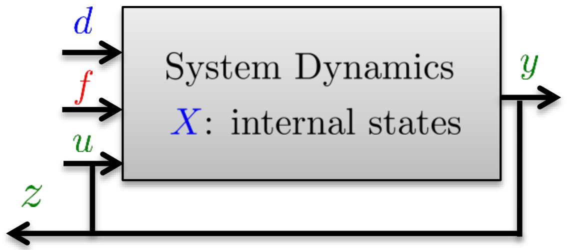
$$x(p)H(p) = 0$$



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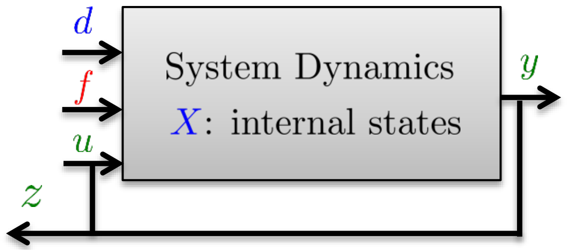

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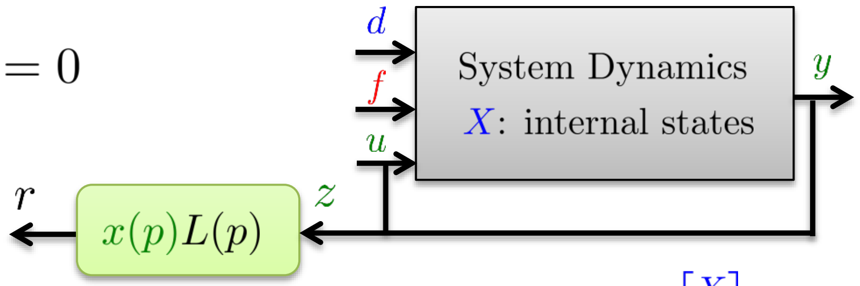

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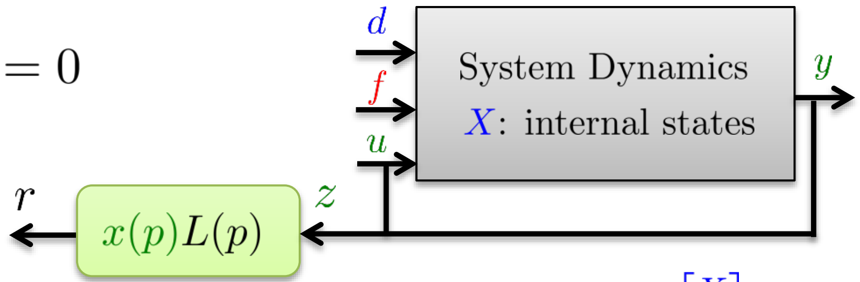

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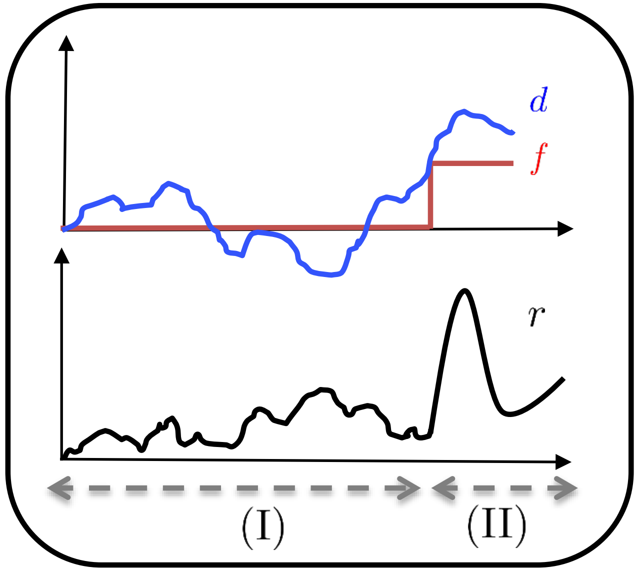

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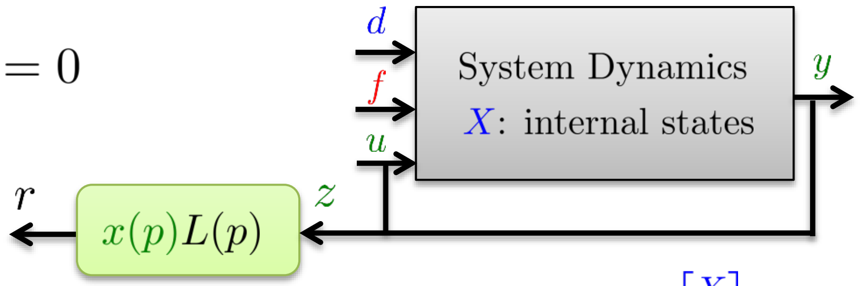

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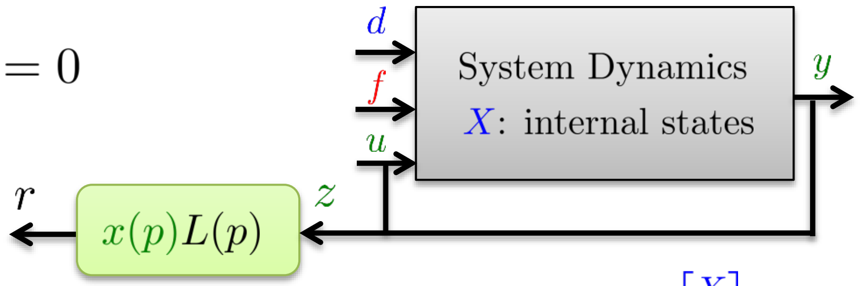

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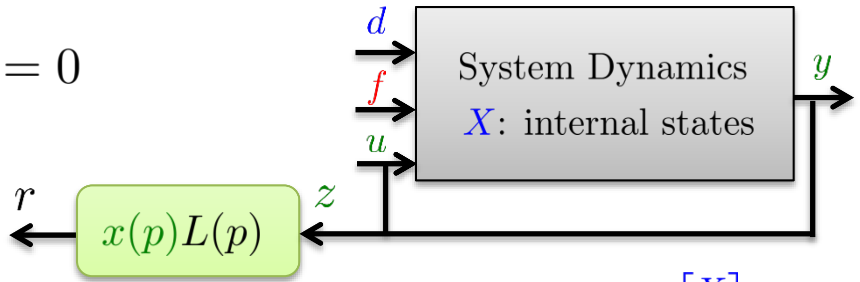

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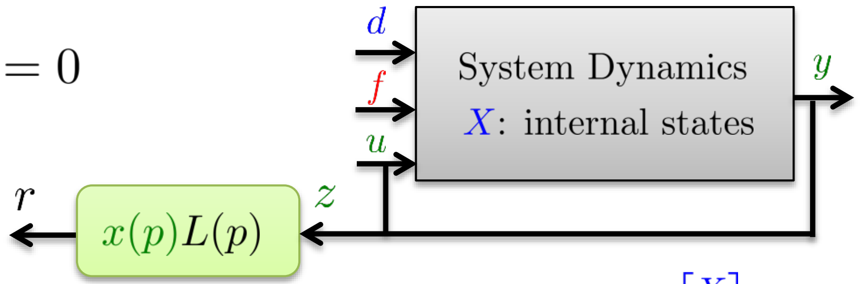

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$$\boxed{Hx = 0} \qquad \boxed{\|(\mathrm{I})\|_{\mathcal{L}_2}^2 = x^\intercal Q(d)x} \qquad \boxed{\|(\mathrm{II})\|_{\mathcal{H}_\infty} = \|Fx\|_\infty}$$

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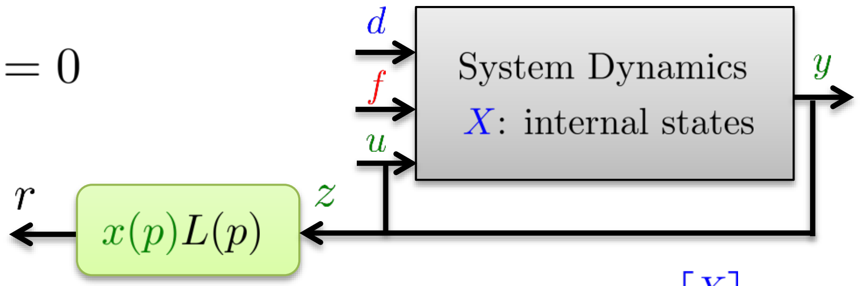

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$$Q(d) : \begin{cases} \text{Nonlinearity of the dynamics} \\ \text{Disturbance information} \\ \text{Linear dynamics} \Rightarrow Q(d) \equiv 0 \end{cases}$$

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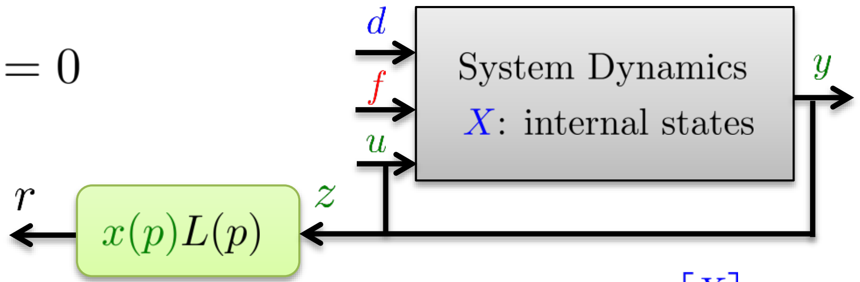

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Sources of difficulty:

(i) Nolinear dynamics

(ii) Unknown disturbance

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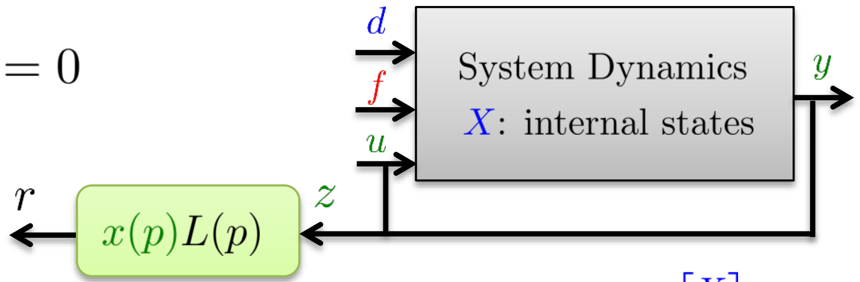

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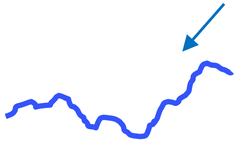
$$x(p)H(p) = 0$$

$$r := \underbrace{x(p)L(p)z}_{\text{Filter}} = \underbrace{-x(p)E(q)}_{(\mathrm{I})} - \underbrace{x(p)F(p)f}_{(\mathrm{II})}$$

$$\boxed{Hx = 0} \qquad \boxed{\|(\mathrm{I})\|^2_{\mathcal{L}_2} = x^\intercal Q(d)x} \qquad \boxed{\|(\mathrm{II})\|_{\mathcal{H}_\infty} = \|Fx\|_\infty}$$

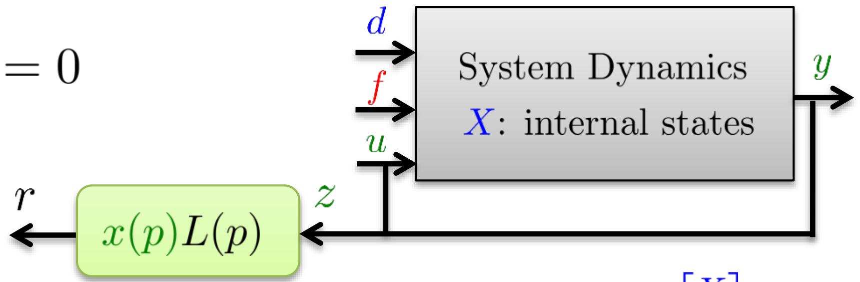
$Q(d) : \{$ Difficult to analyze the map $d \mapsto Q(d)$



$$\begin{bmatrix} * & \cdots & * \\ \vdots & \ddots & \vdots \\ * & \cdots & * \end{bmatrix}$$

Filter Construction

$$\underbrace{E(q)}_{\text{Nonlinear term}} + H(p)q + L(p)z + F(p)f = 0$$



$$x(p)H(p) = 0$$

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$q := \begin{bmatrix} X \\ d \end{bmatrix}$

$z := \begin{bmatrix} y \\ u \end{bmatrix}$

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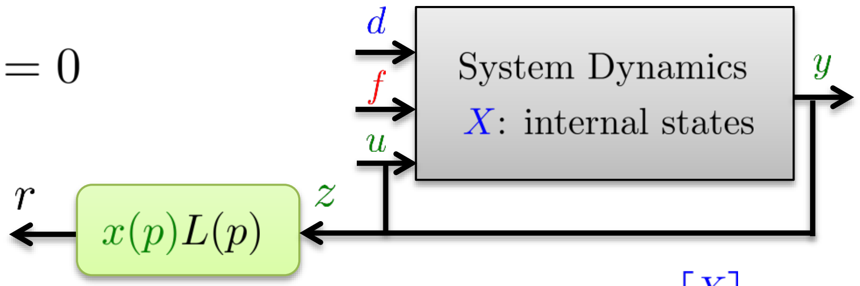
$$\boxed{Hx = 0} \qquad \boxed{\big\|(\mathrm{I})\big\|^2_{\mathcal{L}_2} = x^\intercal Q(d) x} \qquad \boxed{\big\|(\mathrm{II})\big\|_{\mathcal{H}_\infty} = \|Fx\|_\infty}$$

$$Q(d) : \begin{cases} \text{Difficult to analyze the map} \quad d \mapsto Q(d) \\ \text{Easy to compute pointwise} \quad Q(d_i) \end{cases}$$

Filter Construction

$$\underbrace{E(q)}_{\text{Nonlinear term}} + H(p)q + L(p)z + F(p)f = 0$$



$$q := \begin{bmatrix} X \\ d \end{bmatrix} \qquad z := \begin{bmatrix} y \\ u \end{bmatrix} \qquad f := \text{fault}$$

$$x(p)H(p) = 0$$

$$r := \underbrace{x(p)L(p)}_{\text{Filter}} z = \underbrace{-x(p)E(q)}_{\text{(I)}} - \underbrace{x(p)F(p)f}_{\text{(II)}}$$

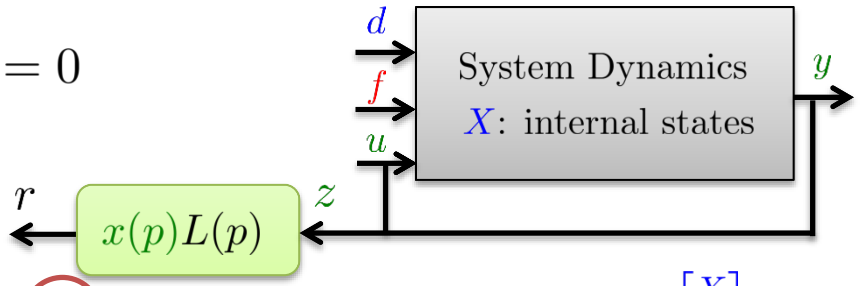
$$\boxed{Hx = 0} \qquad \boxed{\left\|\text{(I)}\right\|_{\mathcal{L}_2}^2 = x^\intercal Q(d)x} \qquad \boxed{\left\|\text{(II)}\right\|_{\mathcal{H}_\infty} = \|Fx\|_\infty}$$

$$Q(d) : \begin{cases} \text{Difficult to analyze the map} \quad d \mapsto Q(d) \\ \text{Easy to compute pointwise} \quad Q(d_i) \end{cases}$$

Learn from data!

Filter Construction

$$\underbrace{E(q)}_{\text{Nonlinear term}} + H(p)q + L(p)z + F(p)f = 0$$



$$q := \begin{bmatrix} X \\ d \end{bmatrix} \qquad z := \begin{bmatrix} y \\ u \end{bmatrix} \qquad f := \text{fault}$$

$$x(p)H(p) = 0 \quad 3$$

$$r := \underbrace{x(p)L(p)}_{\text{Filter}}z = \underbrace{-x(p)E(q)}_{\text{(I)}} - \underbrace{x(p)F(p)f}_{\text{(II)}}$$

1 2

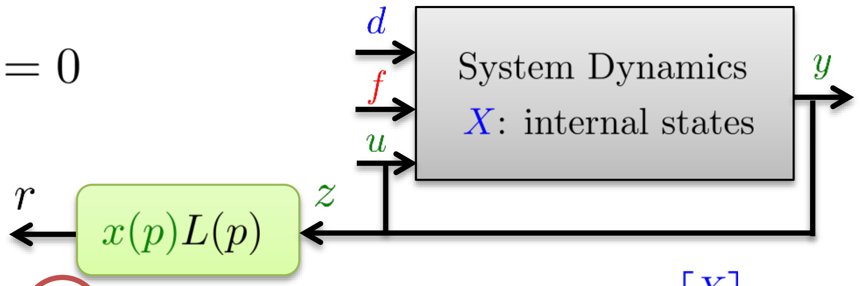
Scenario Program

$$\begin{cases} \min\limits_{x,\gamma} & \gamma \\ \text{s.t.} & x^\intercal Q(d_i)x \le \gamma, \quad \forall i \le N \quad 1 \\ & \|Fx\|_\infty \ge 1 \quad 2 \\ & Hx = 0 \quad 3 \end{cases}$$

$\{d_i\}_{i \le N}$

Filter Construction

$$\underbrace{E(q)}_{\text{Nonlinear term}} + H(p)q + L(p)z + F(p)f = 0$$



$$x(p)H(p) = 0 \quad ③$$

$$r := \underbrace{x(p)L(p)}_{\text{Filter}} z = \underbrace{-x(p)E(q)}_{\text{(I)}} \;①\; - \underbrace{x(p)F(p)f}_{\text{(II)}} \;②$$

$$q := \begin{bmatrix} X \\ d \end{bmatrix}$$

$$z := \begin{bmatrix} y \\ u \end{bmatrix}$$

$f :=$ fault

Scenario Program

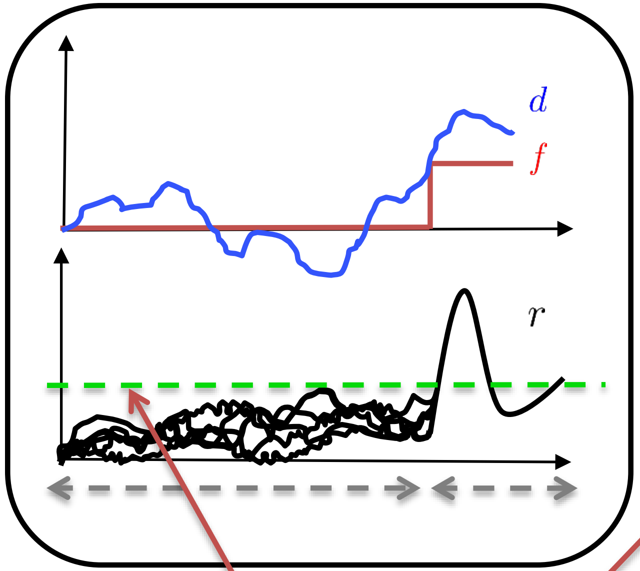
$$\begin{cases} \min\limits_{x,\gamma} & \gamma \\ \text{s.t.} & x^\intercal Q(d_i)x \le \gamma, \quad \forall i \le N \quad ① \\ & \|Fx\|_\infty \ge 1 \quad ② \\ & Hx = 0 \quad ③ \end{cases}$$

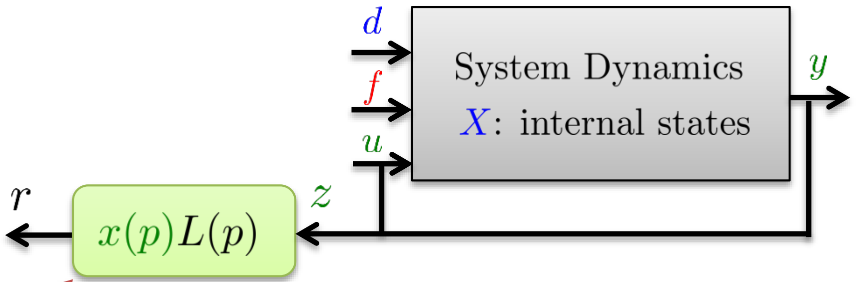
$\{d_i\}_{i \le N}$

Optimizer $x^\star$

Optimal value $\gamma^\star$

Filter Construction





Design parameters:

(i) Filter

(ii) Threshold

Optimizer $x^\star$

Optimal value $\gamma^\star$

Scenario Program

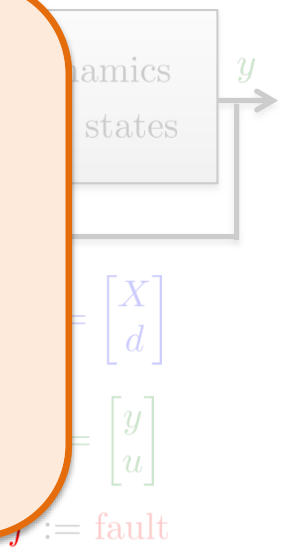
$$\begin{cases} \min\limits_{x,\gamma} & \gamma \\ \text{s.t.} & x^\intercal Q(d_i)x \le \gamma, \quad \forall i \le N \\ & \|Fx\|_\infty \ge 1 \\ & Hx = 0 \end{cases}$$

$\{d_i\}_{i \le N}$

Filter Construction

Model + data
$\Rightarrow$ Convex Optimization

- N: How many?
- For which guarantees?
- Real-time implementation



Scenario Program

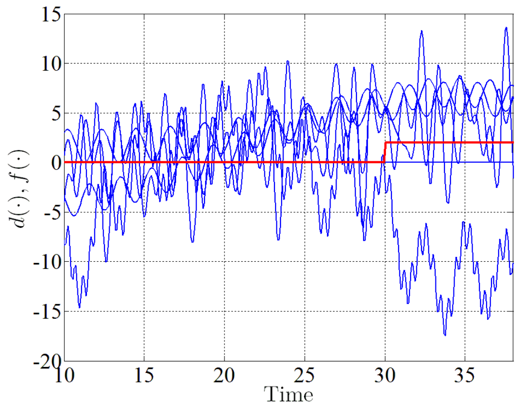
$$\begin{cases} \min\limits_{x,\gamma} & \gamma \\ \text{s.t.} & x^\intercal Q(d_i) x \le \gamma, \quad \forall i \le N \\ & \|Fx\|_\infty \ge 1 \\ & Hx = 0 \end{cases}$$

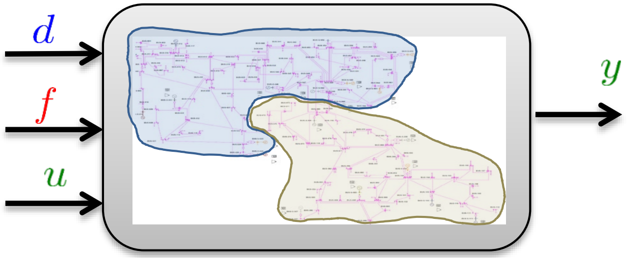
$\{d_i\}_{i \le N}$

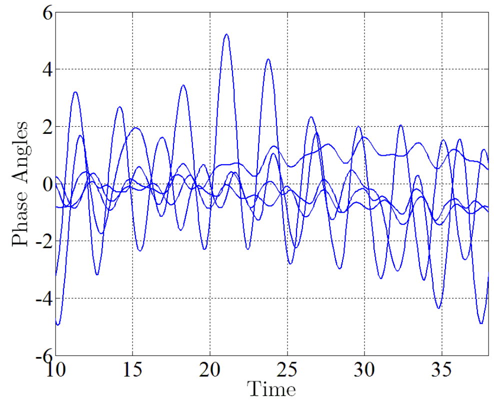
Optimizer $x^\star$

Optimal value $\gamma^\star$

Results







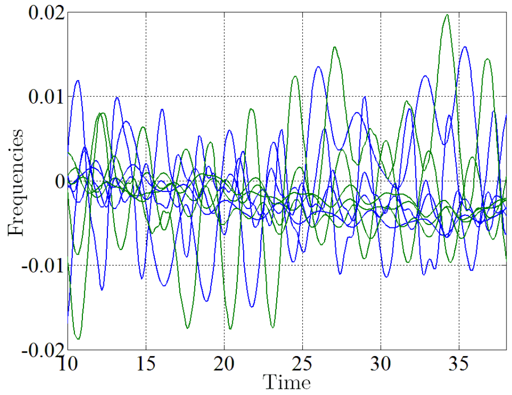
System Dynamics:

✓ ODE: Nonlinear 59 states

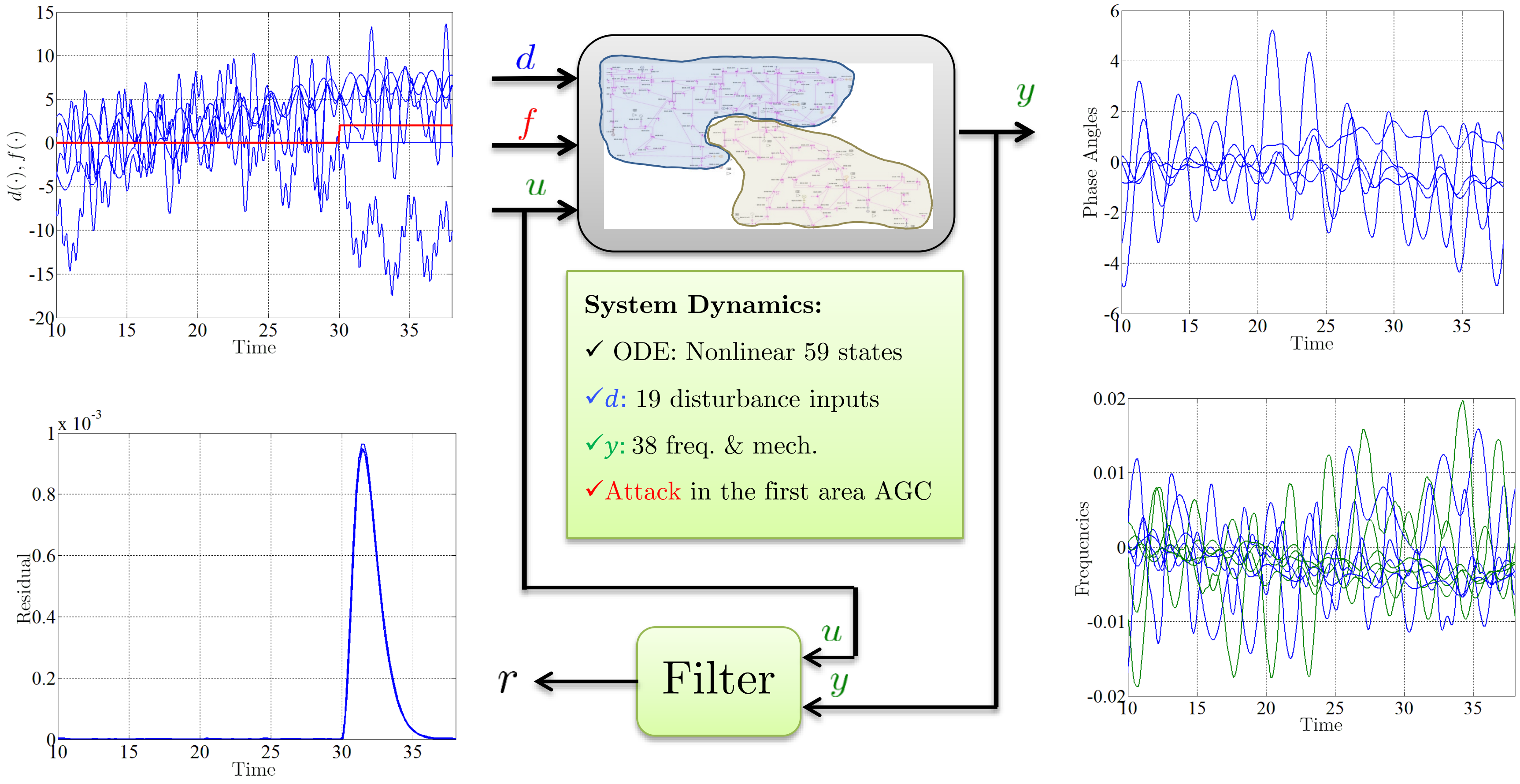
✓d: 19 disturbance inputs

✓y: 38 freq. & mech.

✓Attack in the first area AGC



Results


$d(\cdot), f(\cdot)$
Time
Residual
x 10^-3
d
f
u
y
System Dynamics:
✓ ODE: Nonlinear 59 states
✓d: 19 disturbance inputs
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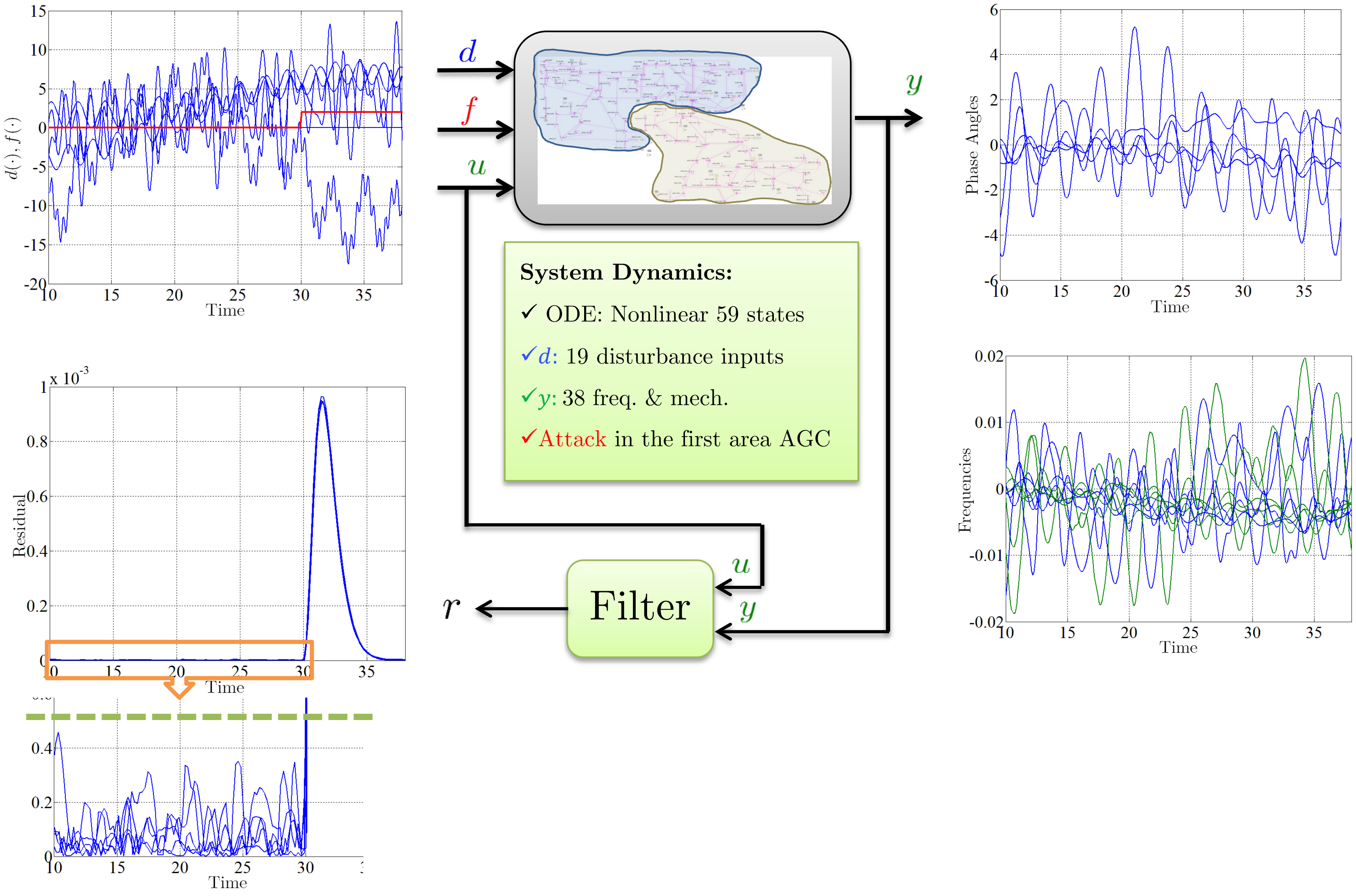
Filter
r
u
y
Phase Angles
Time
Frequencies
Time

Results



$d(\cdot), f(\cdot)$
Time
Residual
x 10^-3
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y
r
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Phase Angles
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Time

Outline

- Anomaly Detection in Dynamic Environment

- Data-driven Decision-Making

Performance Guarantees

Loss function: $\ell(x, d) \in \mathbb{R}$

$x \in \mathbb{X}$
decision

$d \in (\mathbb{D}, \mathbb{P})$
uncertainty

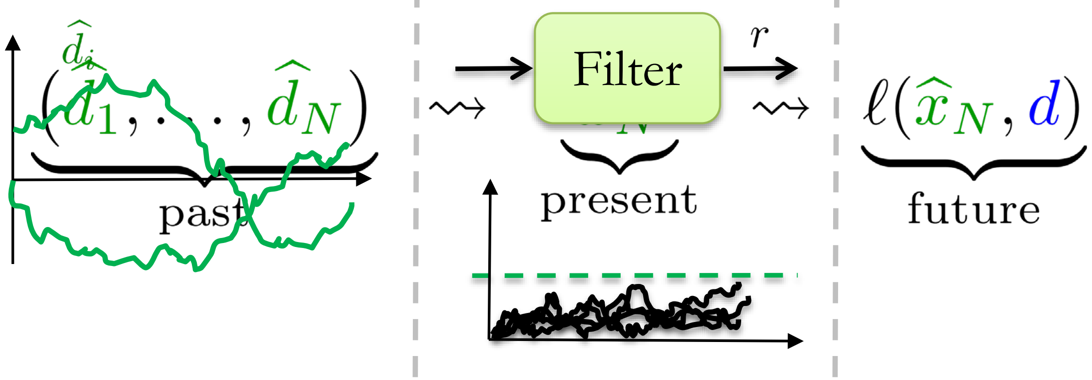
Performance Guarantees

Loss function: $\ell(x, d) \in \mathbb{R}$

$\underset{\text{decision}}{x \in \mathbb{X}}$ $\underset{\text{uncertainty}}{d \in (\mathbb{D}, \mathbb{P})}$

$$\underbrace{(\widehat{d}_1, \ldots, \widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

Performance Guarantees



Performance Guarantees

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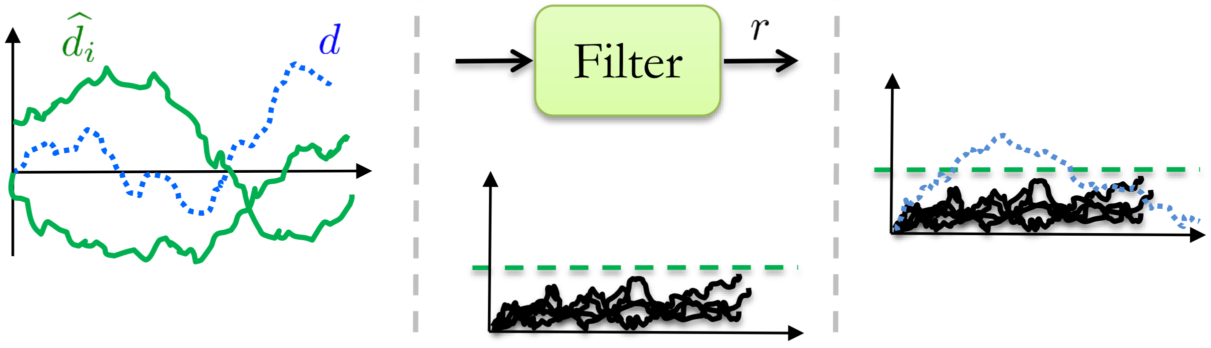

Performance Guarantees

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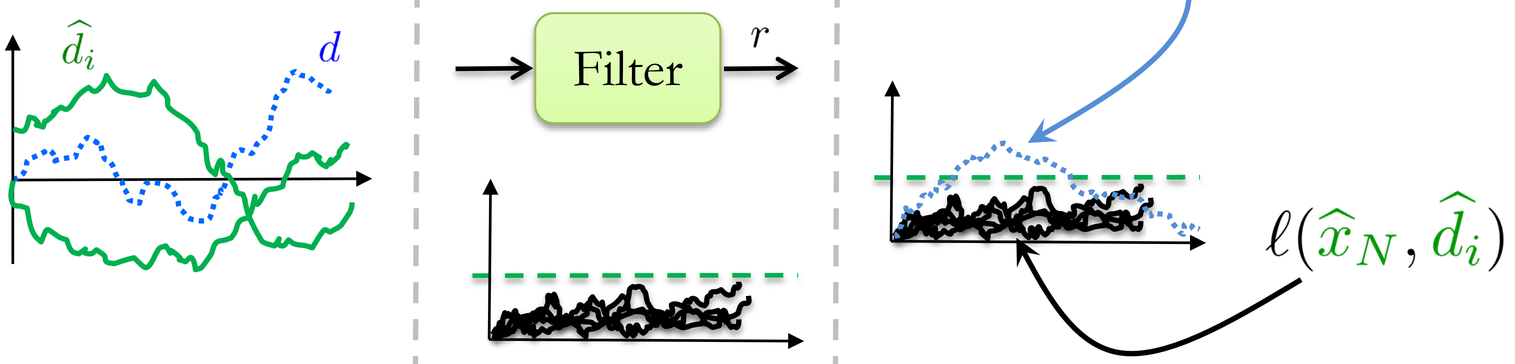
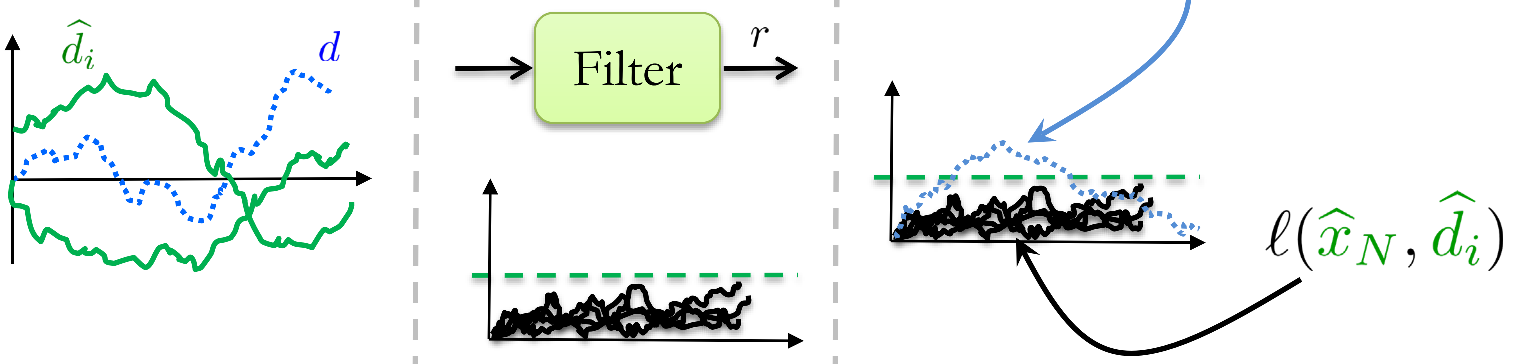


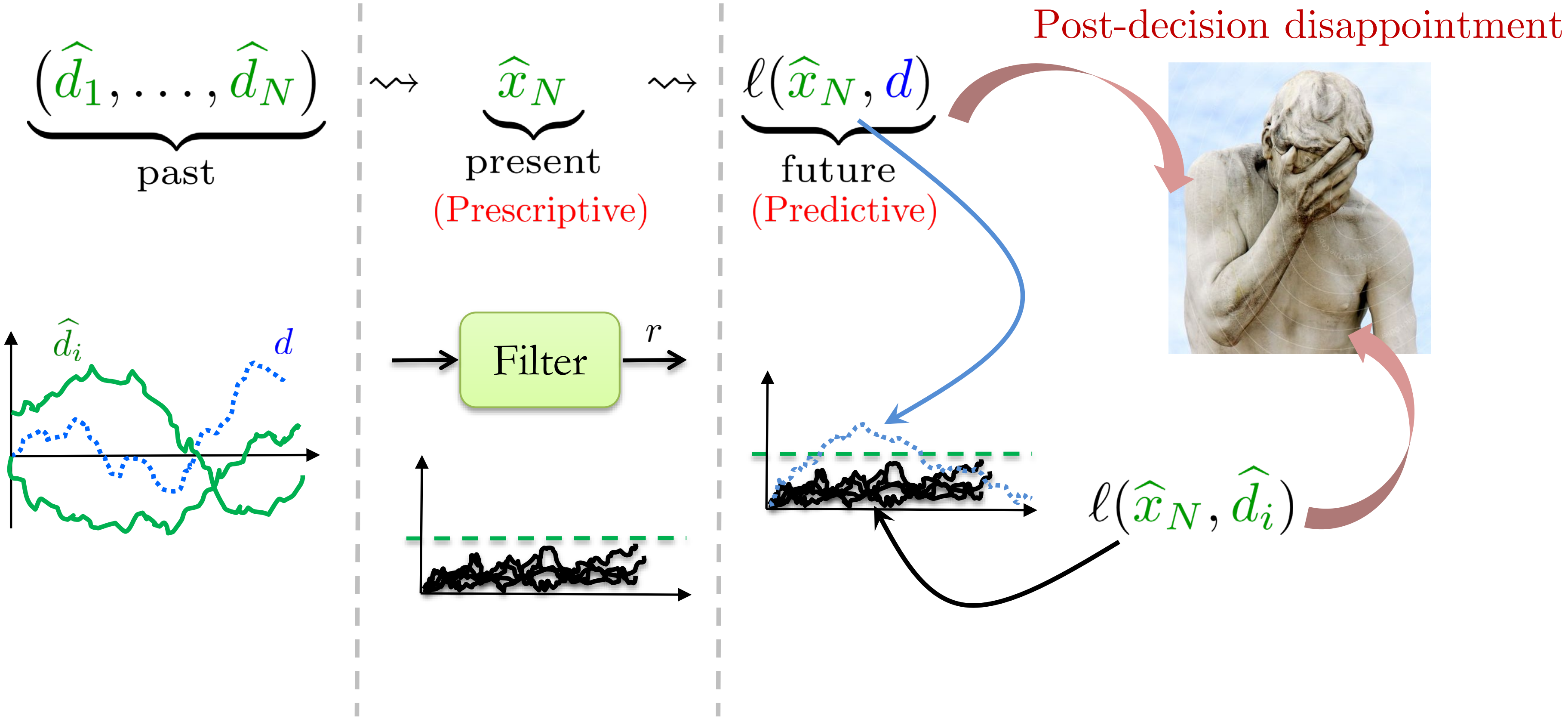
Performance Guarantees

$$\underbrace{(\widehat{d}_1, \ldots, \widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

(Prescriptive) (Predictive)



Performance Guarantees



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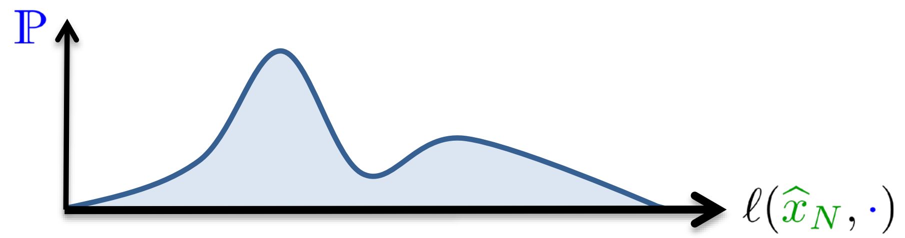

Performance Guarantees

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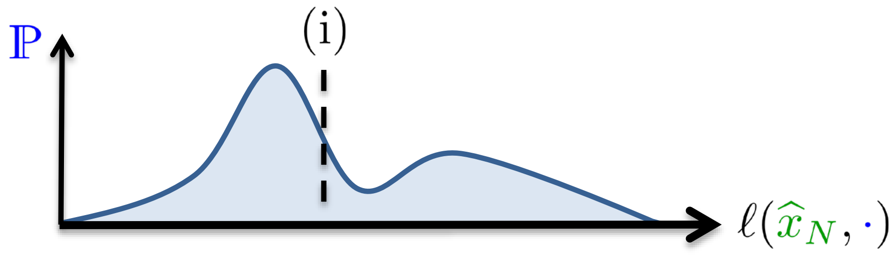
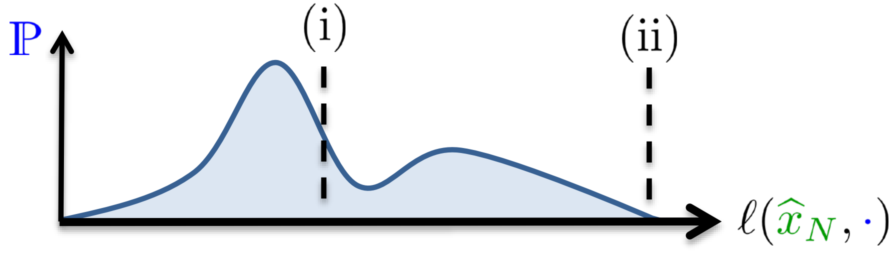


(i) $\mathbb{E}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]$

Performance Guarantees

$$\underbrace{(\widehat{d}_1, \ldots, \widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

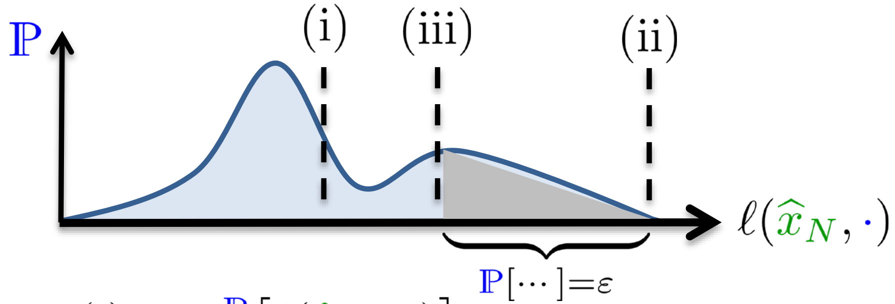


(i) $\mathbb{E}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]$

(ii) $\max_{d \in \mathbb{D}} \ell(\widehat{x}_N, d)$

Performance Guarantees

$$\underbrace{(\widehat{d}_1, \ldots, \widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$



(i) $\mathbb{E}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]$

(ii) $\max_{d \in \mathbb{D}} \ell(\widehat{x}_N, d)$

(iii) $\mathrm{VaR}_{\varepsilon}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]$

Performance Guarantees

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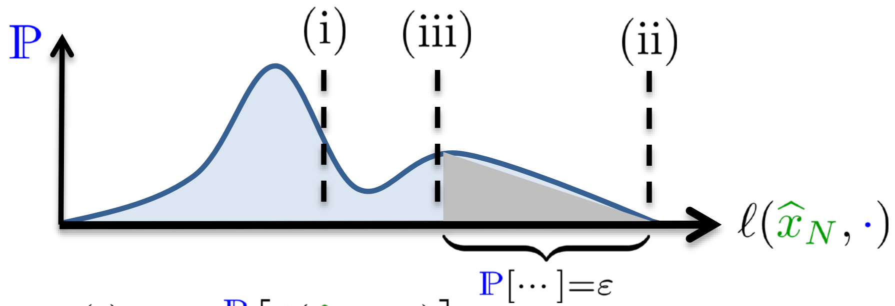

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(ii) $\max_{d\in\mathbb{D}} \ell(\widehat{x}_N, d)$

(iii) $\mathrm{VaR}^{\mathbb{P}}_{\varepsilon}\left[\ell(\widehat{x}_N, d)\right]$

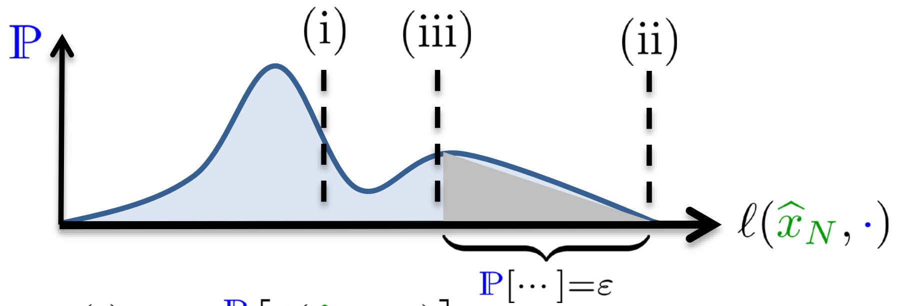
$\mathbb{P}\{\textit{false alarm}\} < \varepsilon$

Performance Guarantees

$$\underbrace{(\widehat{d}_1,\ldots,\widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

Distributionally Robust Optimization

$$\min_{x\in\mathbb{X}} \sup_{\mathbb{Q}\in\widehat{\mathcal{P}}_N} \mathbb{E}^{\mathbb{Q}}[\ell(x,d)]$$



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(ii) $\max_{d\in\mathbb{D}} \ell(\widehat{x}_N, d)$

(iii) $\mathrm{VaR}_{\varepsilon}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]$

Performance Guarantees

$$\underbrace{(\widehat{d}_1,\ldots,\widehat{d}_N)}_{\text{past}} \;\rightsquigarrow\; \underbrace{\widehat{x}_N}_{\text{present}} \;\rightsquigarrow\; \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

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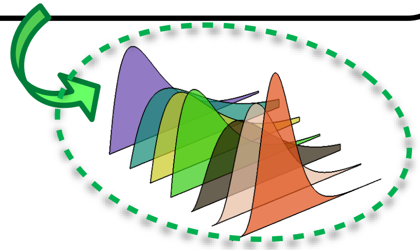
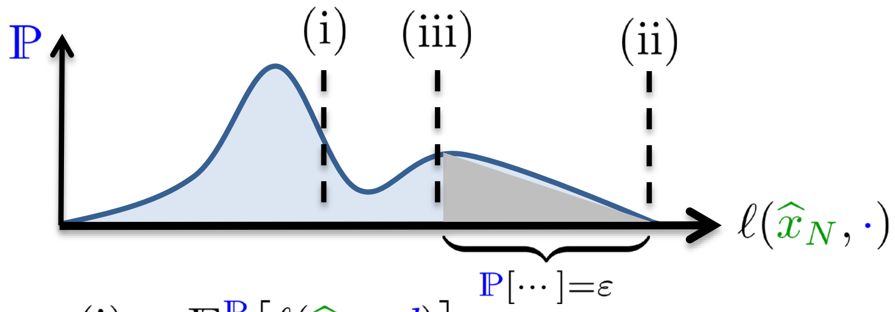
$$\min_{x\in\mathbb{X}} \;\sup_{\mathbb{Q}\in\widehat{\mathcal{P}}_N} \;\mathbb{E}^{\mathbb{Q}}[\ell(x,d)]$$



Ambiguity Set



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(ii) $\max_{d\in\mathbb{D}} \ell(\widehat{x}_N, d)$

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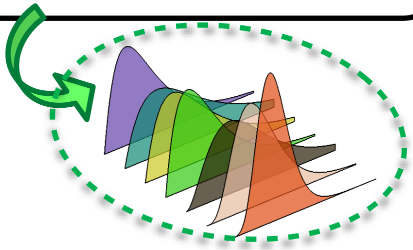
Performance Guarantees

$$\underbrace{(\widehat{d}_1,\ldots,\widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

Distributionally Robust Optimization

$$\widehat{J}_N = \min_{x\in\mathbb{X}} \sup_{\mathbb{Q}\in\widehat{\mathcal{P}}_N} \mathbb{E}^{\mathbb{Q}}[\ell(x,d)]$$



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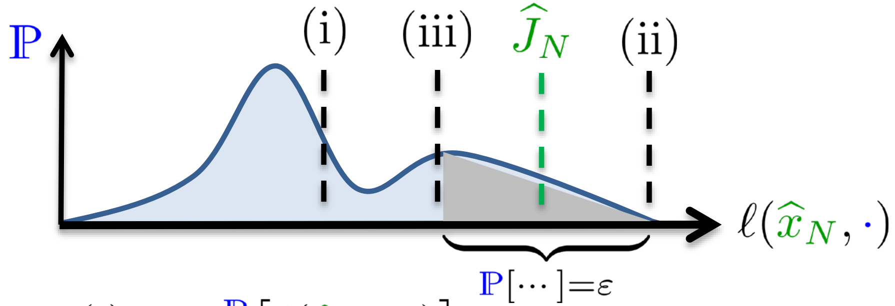

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Performance Guarantees

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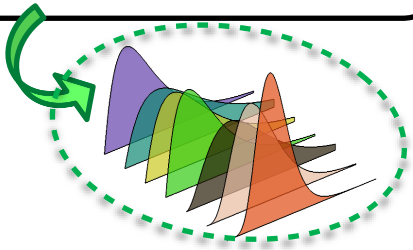

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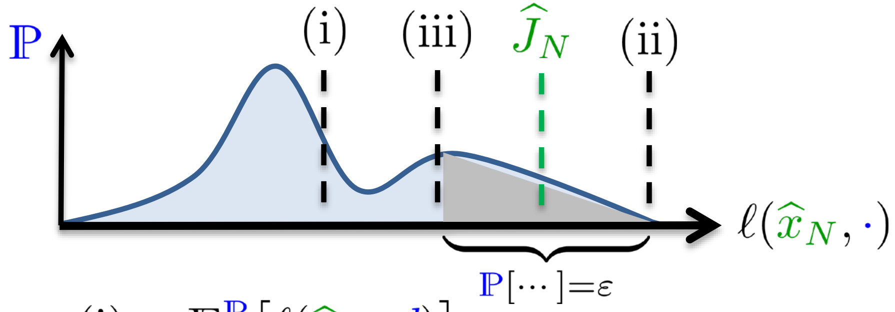

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(iii) $\mathrm{VaR}_{\varepsilon}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]$

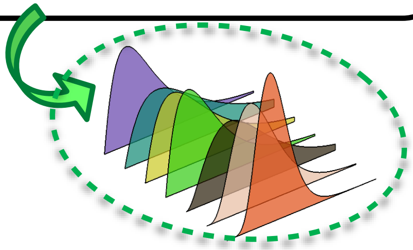
- Statistics: $\widehat{J}_N$ "$\geq$" future performance (i)-(iii)
- Optimization: Is DRO computationally tractable?

Performance Guarantees

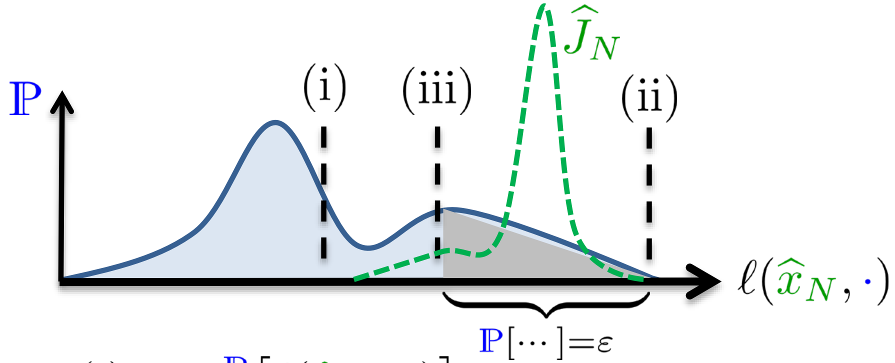
$$\underbrace{(\widehat{d}_1,\ldots,\widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

Distributionally Robust Optimization

$$\widehat{J}_N = \min_{x\in\mathbb{X}} \sup_{\mathbb{Q}\in\widehat{\mathcal{P}}_N} \mathbb{E}^{\mathbb{Q}}[\ell(x,d)]$$



Ambiguity Set



(i) $\mathbb{E}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]$

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(iii) $\mathrm{VaR}_{\varepsilon}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]$

- <u>Statistics:</u> $\widehat{J}_N$ " $\geq$ " future performance (i)-(iii)
- <u>Optimization:</u> Is DRO computationally tractable?

Performance Guarantees

$$\underbrace{(\widehat{d}_1, \ldots, \widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

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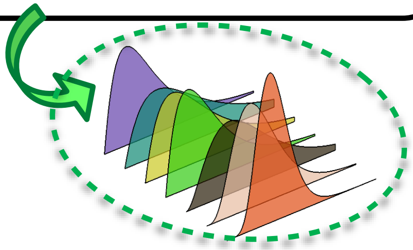
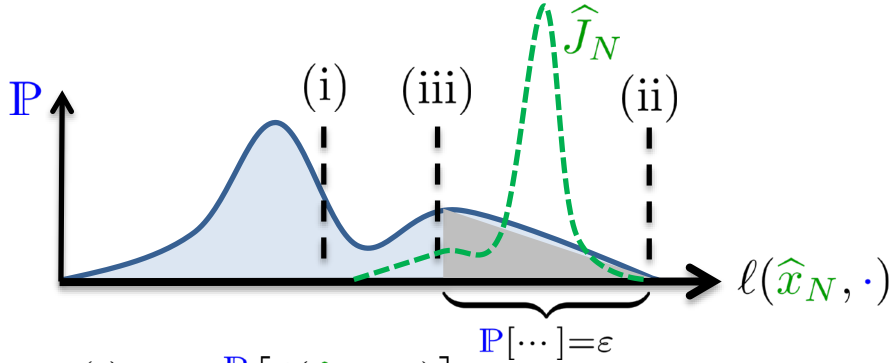


Ambiguity Set



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(ii) $\max_{d \in \mathbb{D}} \ell(\widehat{x}_N, d)$

(iii) $\mathrm{VaR}_{\varepsilon}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]$

- Finite sample guarantee:

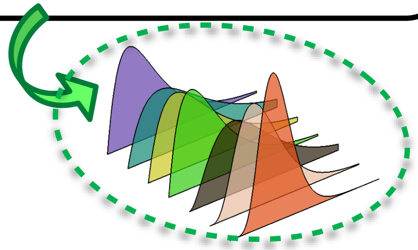
$$\mathbb{P}^N\left[\widehat{J}_N \geq \text{(iii)}\right] \geq 1 - \beta(N)$$

Performance Guarantees

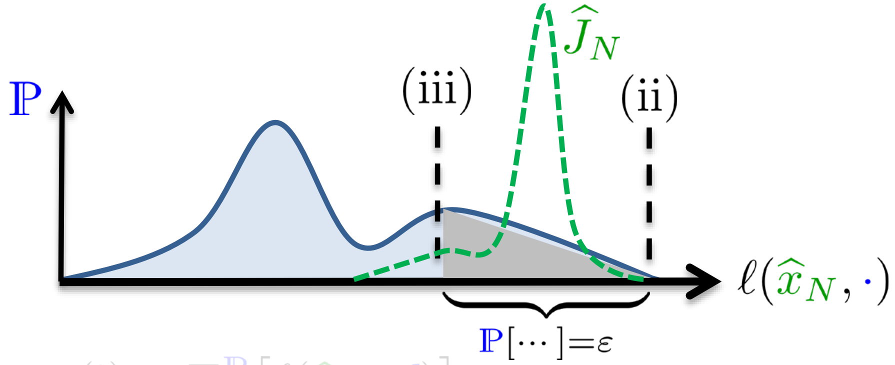
$$\underbrace{(\widehat{d}_1,\ldots,\widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

Distributionally Robust Optimization

$$\widehat{J}_N = \min_{x\in\mathbb{X}} \sup_{\mathbb{Q}\in\widehat{\mathcal{P}}_N} \mathbb{E}^{\mathbb{Q}}[\ell(x,d)]$$



Ambiguity Set
?



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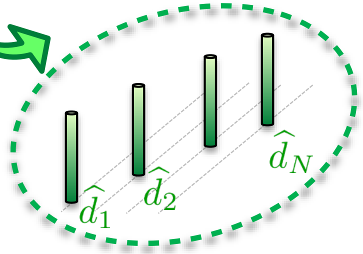
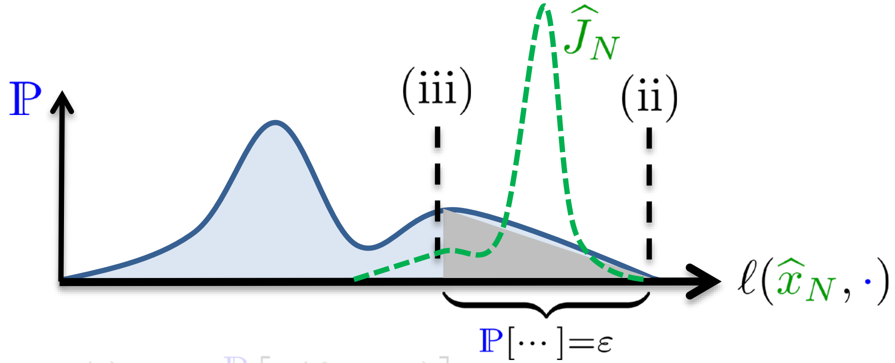


Scenario Program



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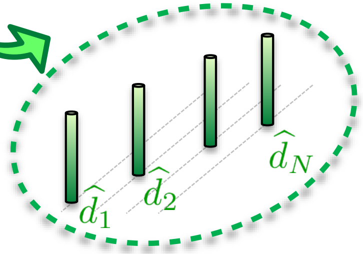

Scenario Program

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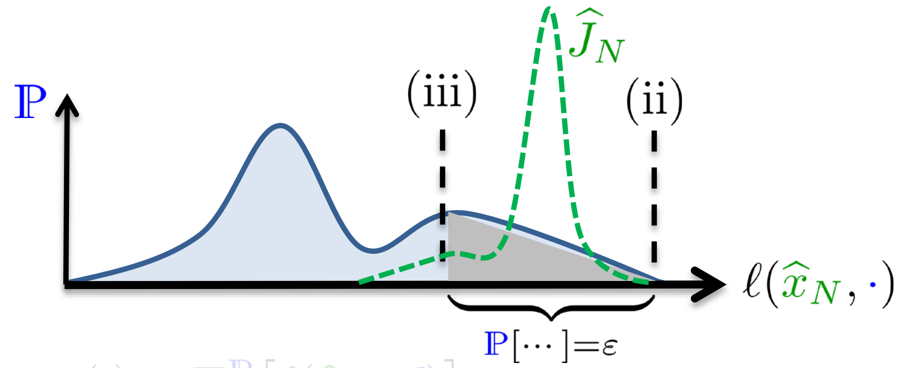

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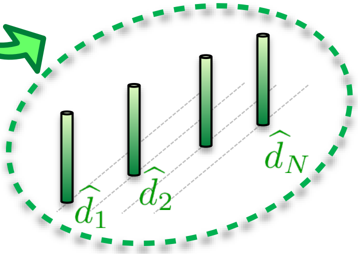

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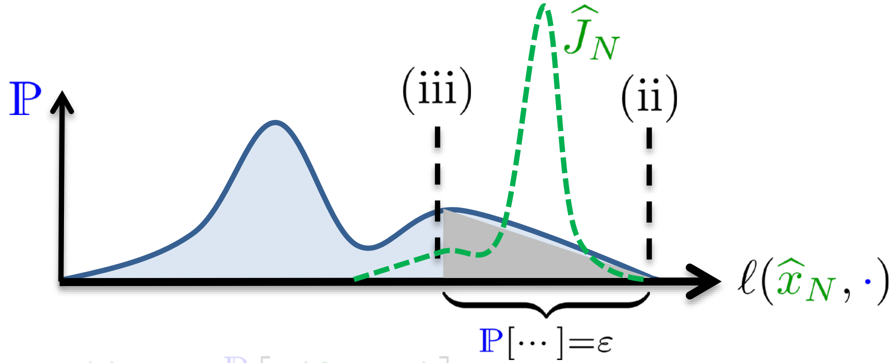

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$\ell(x,d)$ convex in x

Performance Guarantees

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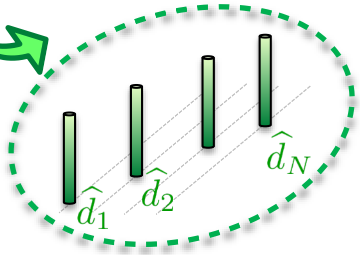

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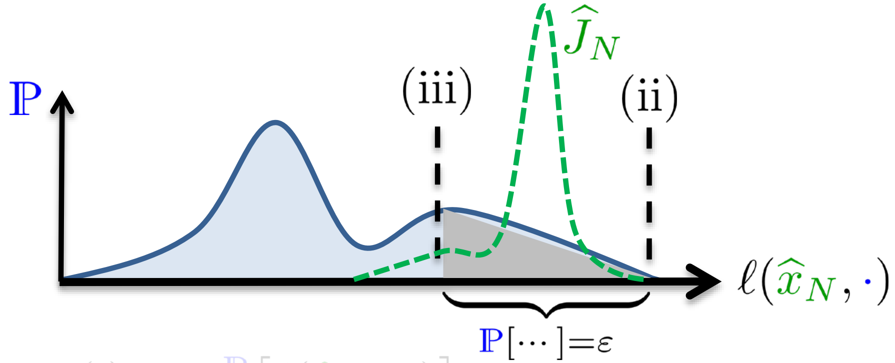

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Statistics meets optimization!

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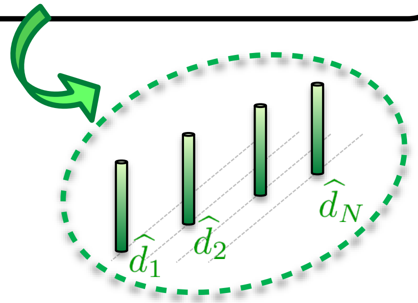
$$\underbrace{(\widehat{d}_1,\dots,\widehat{d}_N)}_{\text{past}} \rightsquigarrow \underbrace{\widehat{x}_N}_{\text{present}} \rightsquigarrow \underbrace{\ell(\widehat{x}_N, d)}_{\text{future}}$$

(1) Totally independent of $\mathbb{P}$

Distributionally Robust Optimization

$$\widehat{J}_N = \min_{x\in\mathbb{X}} \ \sup_{\mathbb{Q}\in\widehat{\mathcal{P}}_N} \ \mathbb{E}^{\mathbb{Q}}[\ell(x,d)]$$



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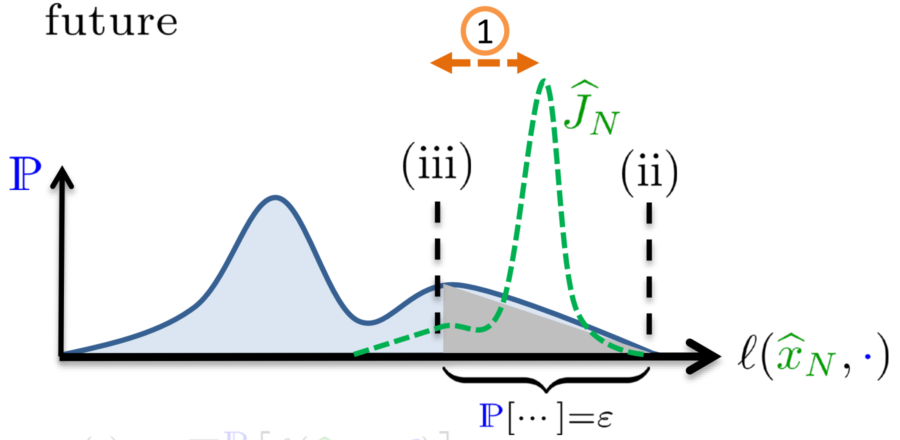

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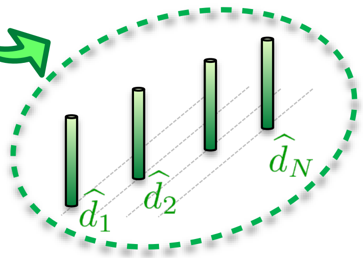

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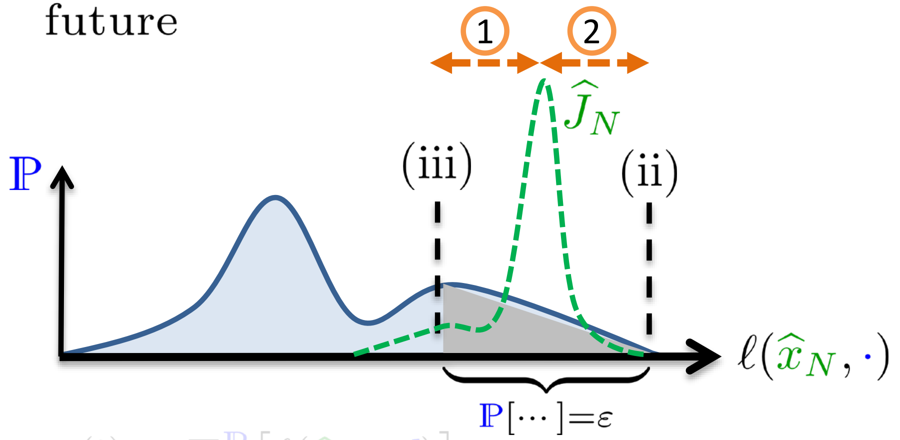
$$\begin{cases} (1) \text{ Totally independent of } \mathbb{P} \\ (2) \text{ Exponentially bad in } \dim(d) \end{cases}$$



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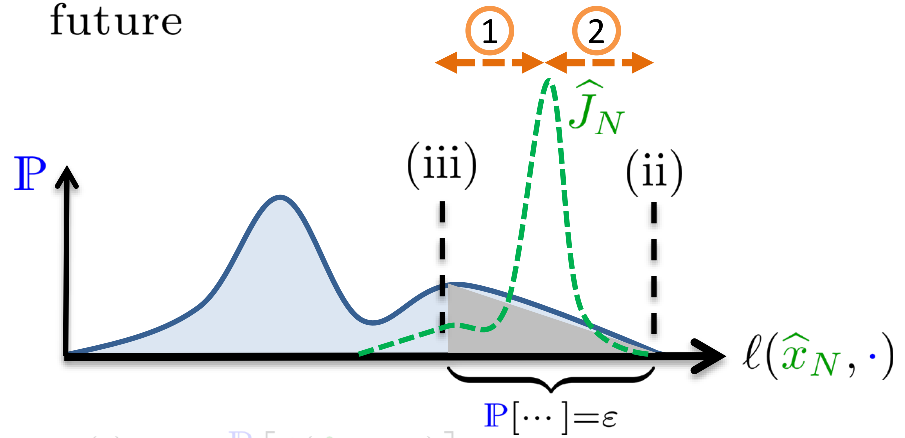

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Real-time implementation

$\widehat{x}_{N+1} \approx \mathrm{Alg}(\widehat{x}_N, \widehat{d}_{N+1})$

- Online convex optimization
- Regret bounds

$\ell(x, d)$ convex in x

Statistics meets optimization!

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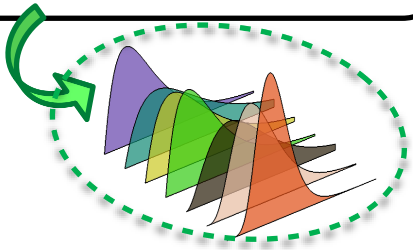
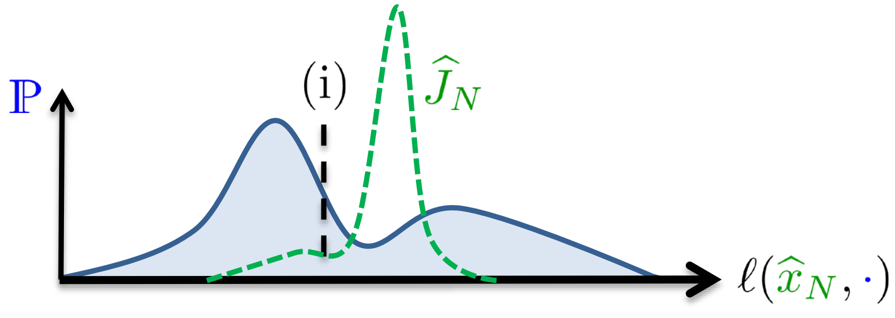


Ambiguity Set



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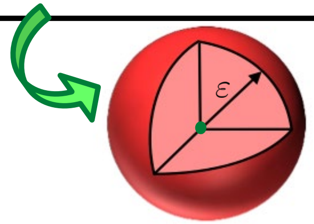

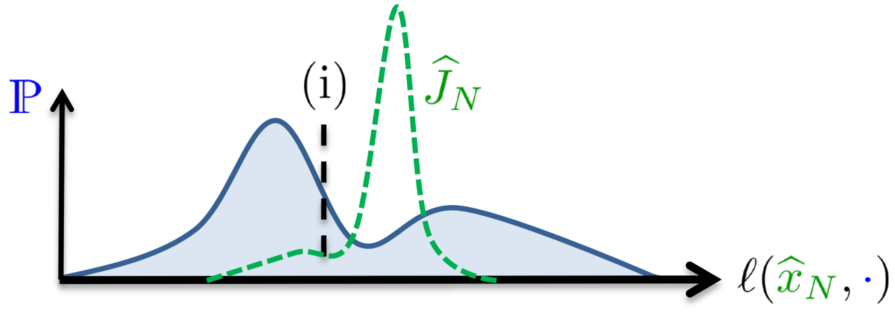

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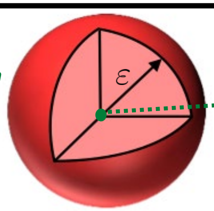
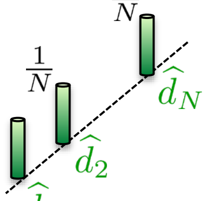
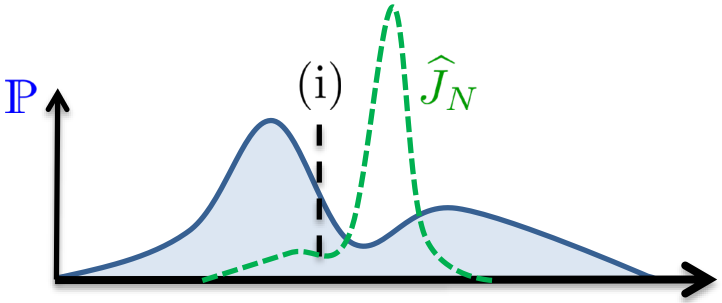


$\widehat{\mathbb{P}}_N$:
empirical distribution





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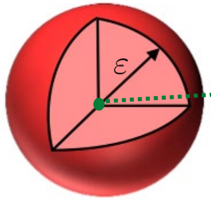

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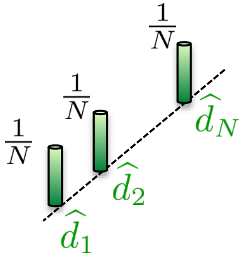
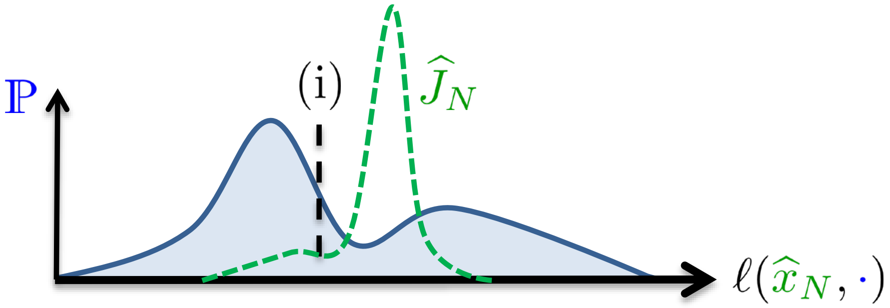


Remarks:

- $\mathbb{P} \in \mathbb{B}_\varepsilon(\widehat{\mathbb{P}}_N) \implies \widehat{J}_N \geq \underbrace{\mathbb{E}^{\mathbb{P}}\left[\ell(\widehat{x}_N, d)\right]}_{\text{(i)}}$
- $\widehat{J}_N = \min_{x \in \mathbb{X}} \frac{1}{N} \sum_{i=1}^{N} \ell(x, \widehat{d}_i) + \varepsilon \|x\|$



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Applications

Security of Power Systems

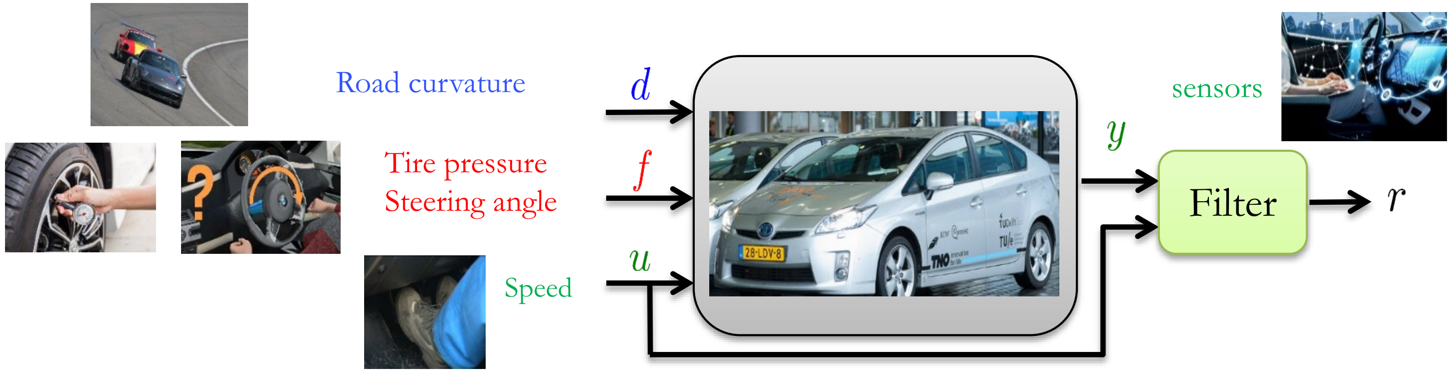
- Deploy the filter in ABB Network Manager [1]
- High-fidelity model
  - Multivariate attacks, game-theoretic setting [2]
  - Hardware in the loop [3]

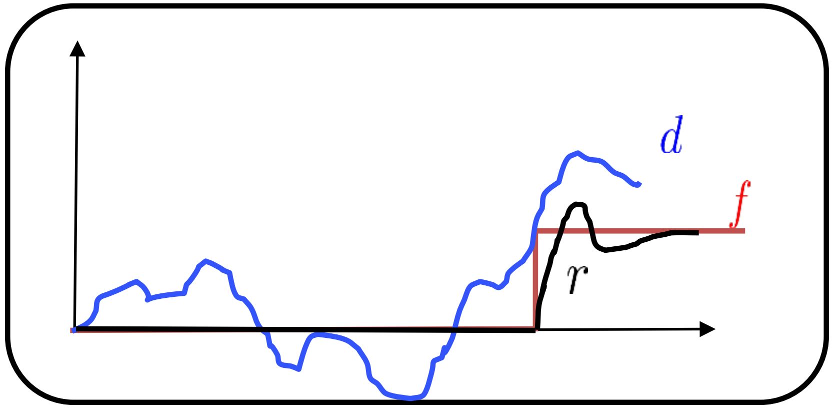
[1] PME, Lygeros, *IEEE Transactions on Automatic Control*, 2016
[2] Pan, Palenski, PME, *IEEE Transactions on Power Systems, 2020*
[3] Pan, Palenski, PME, *IEEE Transactions on Smart Grid, 2021*

Applications

Autonomous vehicles

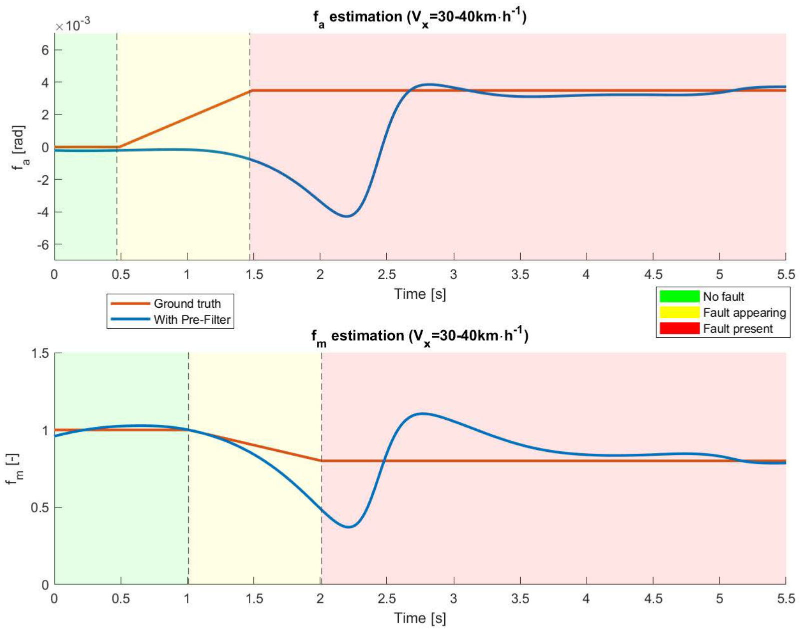
- Multiple faults [1]
- From detection to estimation [1]

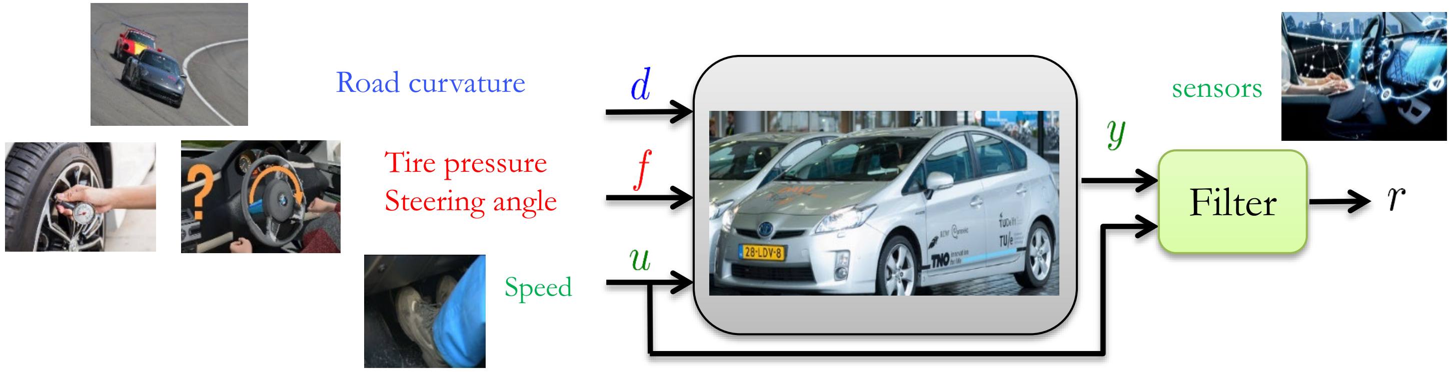


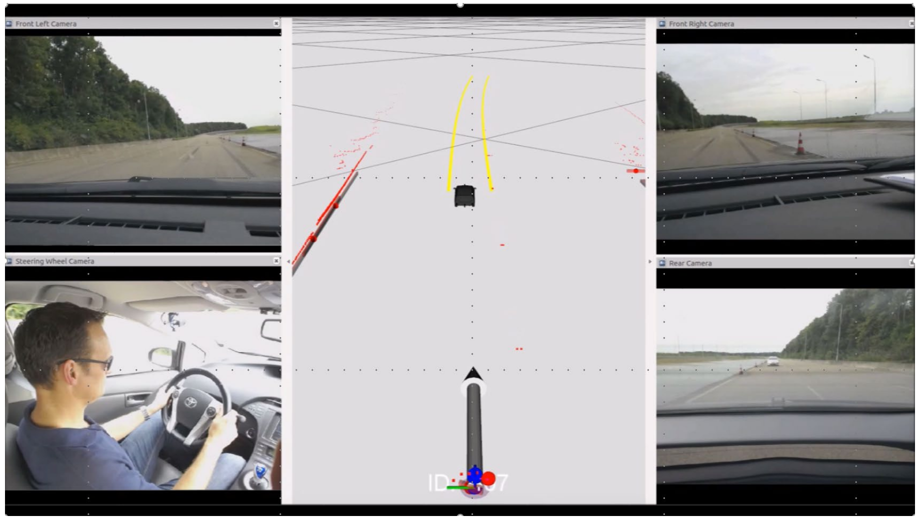


[1] C. van der Ploeg, M. Alirezaie, N. van de Wouw, PME, *IEEE Transactions on Automatic Control (TAC)*, minor revision, 2022

Applications





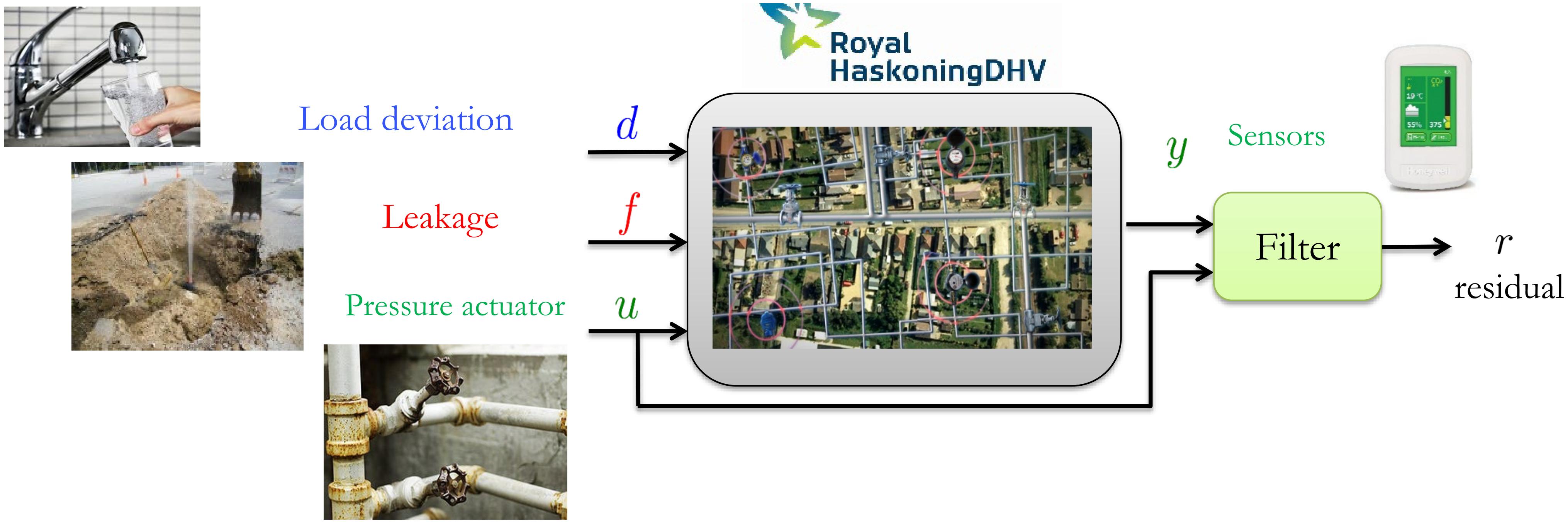


[1] C. van der Ploeg, M. Alirezaie, N. van de Wouw, PME, *IEEE Transactions on Automatic Control (TAC)*, minor revision, 2022

Applications

Water distribution network

- Active detection [1]



[1] G. van Lagen, E. Abraham, PME, *IEEE Transactions on Control Systems and Technology (TCST)*, 2022

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Dynamic anomaly detection

- Ploeg, Alirezaie, van de Wouw, and **PME**, “Multiple Faults Estimation in Dynamical Systems: Tractable Design and Performance Bounds”, *IEEE Transaction on Automatic Control (TAC)*, minor revision, 2022
- **PME** and Lygeros, “A Tractable Fault Detection and Isolation Approach for Nonlinear Systems with Probabilistic Performance”, *IEEE Transaction on Automatic Control (TAC)*, 2016

Data-driven Analytics

- **PME** and Kuhn, “Data-Driven Distributionally Robust Optimization using the Wasserstein Metric: Performance Guarantees and Tractable Reformulations”, *Mathematical Programming*, 2018
- **PME**, Sutter, and Lygeros, “Performance Bounds for the Scenario Approach and an Extension to a Class of Non-convex Programs”, *IEEE Transaction on Automatic Control, 2015*
- Nguyen, Kuhn, **PME,** “Distributionally robust inverse covariance estimation: The Wasserstein shrinkage estimator”, *Operations Research (OR)*, 2021

Applications: Power systems Water distribution

- Pan, Palenski, and **PME**, “From Static to Dynamic Anomaly Detection with Application to Power System Cyber Security”, *IEEE Transactions on Power Systems*, 2021
- G. van Lagen, E. Abraham, and **PME**, “A Bayesian Approach for Active Fault Isolation with an Application to Leakage Localization in Water Distribution Networks”, *IEEE Transaction on Control Systems and Technology*, 2021

Fast dynamic programming

- Kolarijani, Max, and **PME**, “Fast Approximate Dynamic Programming for Input-Affine Dynamics”, *NeurIPS*, 2021